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Mathematical and Architectural Concepts Manifested in Iannis Xenakis's Piano Music

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Mathematical and Architectural Concepts Manifested in Iannis Xenakis's Piano Music

by

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To mom and dad,

Soomin, Bosco,

And Marcel,

For their endless support and love

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This treatise explores the principles of a figure who was equally versed in several disciplines: music, architecture, and mathematics. Such combination has permitted the possibility of a great expansion of aesthetic principles, techniques, and general concepts of musical coherence beyond those of an ordinary composer.

During the twentieth century, interest in new musical sources flourished. Composers explored music in combination with other disciplines; composers developed serialism, used electronic devices, and researched other fields such as mathematics and architecture.

Iannis Xenakis (1922 – 2001) is such a figure. A Greek composer, who resided in France, he keenly sought a way to approach the creation of music from

a different field in order to expand the possibilities of composition, and used computer and other electronic devices as part of his overall techniques. Being a civil engineer and an architect himself, Xenakis related music to architecture and mathematics.

The probability theory forms the macrostructure and the distribution of notes in Xenakis's music. From the structure to such distribution, the probability/stochastic laws take the main role in making decisions. Other factors, such as mathematical set theory, are also used, for example in his solo piano piece, *Herma*.

Xenakis's architectural constructions also influenced the main concepts and structures of his compositions. Projects such as the Couvent de la Tourette and the Phillips Pavilion show keen relationship with the composition of *Metastaseis* and *Concret P.H.*

Golden proportion, one of the main considerations in architecture and a proportional phenomenon in nature, has served as a formulaic source for many composers of the twentieth century. In Xenakis's *Herma* and *Evryali*, golden mean underlies the macro- and the microstructure. The mathematical set theory in *Herma* and the arborescence method in *Evryali* are joined with golden section to determine the actual deployment of notes, the occurrence of phrases, and durations.

Despite the difficulties and the discussions on the performance of Xenakis's music, it seems that many performers find Xenakis's pieces challenging, yet interesting. The mathematical and symbolic pre-compositional principles do not seem directly relevant to music. However, they intellectually underlie the construction of the pieces and show inevitable associations with music.

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CHAPTER 1
Introduction

At the start of the twentieth century, interest in new musical sources flourished. Spurred by the second Viennese school, the twelve-tone series prevailed and this principle was extended to the other parameters of music, such as timbre, duration, and dynamics. This resulted in total serialism, which led to composition or performance with the necessary assistance of electronic devices and has been referred to as integral serialism, total control, ultra-rationality, all of which refer in part to the traditional notion of sequential order, but more fundamentally refer to the association of parameters to a single event, for example, a note. It is not only that composers searched for tools to create new sounds or to aid in composition, but also that they looked for new sources of compositional method and organization leading to a system. Some composers actively used the electronic medium to reproduce their compositions. Others approached it differently, by using the electronic or computer devices not only as a medium to reproduce the sound but also as pre-compositional or compositional aid.

Iannis Xenakis (1922 – 2001), a Greek composer, who resided in France, is one of the main figures who keenly sought for a way to approach the creation of music from a different field to expand the possibilities of composition, and used computer and other electronic devices as part of his overall techniques. Being a civil engineer and an architect himself, Xenakis deeply related music to architecture and mathematics. Milton Babbitt (b. 1916) applied some mathematical principles for the purpose of serial development. However, Xenakis's method was more complicated and more geared toward pure mathematics, which is rooted in his civil engineering work.

The stochastic laws[1] form the macro structure and determine micro distribution of details in Xenakis's music. He subdivided the compositional process into three stages, which he related to mathematical practice: (1) outside-time algebra, (2) temporal algebra, and (3) in-time algebra. According to Xenakis's explanations, the outside-time algebra refers to elements that are decided without consideration of sequence, temporal algebra to sequential organization of elements excluding actual events, and the in-time algebra to the application of outside-time elements to temporal sequence.[2] From the structure to the distribution of notes in the structure, the probability laws are the primary determinant for making decisions. Other factors, such as mathematical set theory,

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[1] Stochastic theory shows the probability that the more a performance is executed, the more determinate the outcome becomes. More explanation will follow in chapter 3.

[2] Detailed explanations to follow in chapter 3.

are also used, for example in his solo piano piece, *Herma*, which was composed in 1961 and commissioned by pianist Yuji Takahashi.

Xenakis's architectural constructions also influenced the main concepts and structures of his musical compositions. The structure and compositional elements of his architectural projects are reflected in the pieces such as *Metastaseis*, an orchestral piece composed in 1954, and *Concret P.H.*, a tape music composed in 1958. The Couvent de la Tourette and the Philips Pavilion are among his architectural designs that affected his compositions most: the design of the Couvent de la Tourette influenced the structure of *Metastaseis*, and the Philips Pavilion, which shares the stochastic principle with *Metastaseis*, gave birth to *Concret P.H.*[3]

Golden proportion (golden mean, golden section, golden ratio) is one of the main considerations in architecture as well as in natural phenomena; the ratio of 0.618 to 1, the golden mean, which will be discussed in the following chapter, is found mostly in the growth of natural objects, and has been studied in constructions and the arts for thousands of years. Early 20th composers, such as Debussy and Bartok, used the golden mean in composition.[4] The structure of

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[3] Further explanation will follow in chapters 2 and 3.

[4] See Erno Lendvai, *Bela Bartok: An Analysis of His Music* (London: Kahn & Averill, 1971)and Roy Howat, *Debussy in proportion* (New York : Cambridge University Press, 1983).

pieces, the occurrence of climax, and further details such as phrasing are often based on the golden section. It is very possible to assume that the golden ratio affected the structure of Xenakis's pieces, for his compositional works are much influenced by the architecture and he, as a Greek descendant, was very well aware of Greek architecture, where golden proportion plays a most crucial role.

In the following chapters, the mathematical principles, equations, and processes relevant to Xenakis's piano music will be reviewed. His architectural works will also be discussed in relation to his music. Two of his piano pieces, *Herma* and *Evryali* (1973), will be closely examined based on the mathematical and architectural methods of his compositions. It is necessary to discuss the principles of Boolean algebra for the structural analysis of *Herma*, for the mathematical set theory formed both macro- and microstructures of the piece. The distribution of sets and their elements (notes) and the actual deployments in the piece will be studied. The occurrence and specific shapes of the arborescence in *Evryali* will also be discussed;[5] *Evryali* is the first piece that Xenakis composed based on the arborescence principle and constructed on the appearance of arborescences.

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[5] According to Xenakis, any given curvy line may reproduce itself so it becomes a bush or a tree shape [arborescence], and the arborescence is placed on pitch versus time space. The initial curvy line or a group of curvy lines might be transformed, through rotation, inversion, retrograde, or their combination, in music.

It seems possible that both *Herma* and *Evryali* will show proportional relevance, such as golden mean, for Xenakis explicitly adopted this proportional principle, the *Modulor*,[6] from Le Corbusier, and applied it to his architectural designs and compositions. For his piano pieces, Xenakis did not express that he used golden proportional elements intentionally. However, it is more likely that he was familiar with golden section principle since he was very well aware of Greek architecture, whose proportional principle is golden section, and was an architect himself, who actively used golden section in his architectural works. Therefore, this study will closely examine the use of golden ratio in *Herma* and *Evryali*: this will be related to the durational proportions, densities,[7] the theoretical and actual deployments, the occurring moment of phrases and others. The study of golden section will help composers see another possibility of constructing music, performers to better understand the structure of the pieces, music historians to find the use of golden section in compositions of this period of time.

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[6] Le Corbusier came up with his modulor system, which was his own regulating principle for his constructions derived from classical proportion and balance and was largely based on the golden section. He used the modulor as a guide of design process. *The Modulor* was published in 1950 and *The Modulor* II in 1955.

[7] The ratio of the number of events to a given measure unit, such as sounds/second. Further explanation will be given in the following chapter.

The issues of performance practice will also be discussed after compositional aspects are examined. Scholars and performers have questioned the accuracy of the performance because of the difficulty and complexity of the pieces. The suggestions for substitution of execution and their effectiveness will be discussed.

CHAPTER 2

Xenakis's Background and the Development of Compositional Methods

Iannis Xenakis was born on May 29, 1922, in Brăila, Romania, to a Greek business family. He was sent to Greece in 1932 with his brothers, to go to a boarding school in Spetse, and he engaged himself in English, French, astronomy, Byzantine liturgical music and especially science and Greek literature, which became his lifelong interests. As a member of the school choir, he became acquainted with Greek and other church music including that of Palestrina,. Despite his early musical experiences through Romanian folk music, Gypsy music, Greek folk music, and Byzantine church music, he tried to get away from these influences; it was his belief that music should not be related to subjective feelings and experiences,[8] and Gypsy or folk music of his childhood evoked very sad memories of his mother who died when Xenakis was five years old.[9]

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[8] Bálint Varga, *Conversations wth Iannis Xenakis* (London: Faber and Faber, 1996), 10.

[9] Ibid., 8.

During adolescence, he recognized his interest in music and decided to learn the piano. He studied solfeggio, notation, and singing at school and learned to play the piano in Romania where he spent his holidays. He also had great interest in science such as physics, simultaneously.

After graduating from secondary school, his father offered him a chance to go to England to study naval engineering; however, he rejected this offer because of his patriotism for Greece. Note that Greece before the Second World War was going through the growing international tension between Germany – Italy Axis, and France – England alliance. Instead, he moved to Athens, preparing to enter the Athens Polytechnic, and continued taking piano and music theory lessons. He passed the entrance examination at the Polytechnic school in 1940. Soon after his study at the Polytechnic had started, he also joined the Resistance against the German invasion in Greece, in 1941, when the German and the British troops were trying to take control of Greece;[10] for these reasons, despite his interest in music, Xenakis had to put music aside for a couple of years. Towards the mid-40s, however, Xenakis became acquainted with a nephew of the General Secretary of the Communist Party who was constantly composing music and trying to find a piano to play even

[10] For this reason, it took seven years for him to graduate from the Athens Polytechnic.

during the extreme turmoil of war. Through him, Xenakis got to know the music of Debussy, Ravel and Bartok for the first time.[11]

During a battle in which citizens and students were involved in December 1944, Xenakis was badly injured and the left side of his face collapsed. Since then, his bright and energetic character had turned to that of a sullen, violent and angry man who was destroyed and embittered.[12] This, later, might have affected his attitude towards the teachers of his music study, such as Honegger and Milhaud. Xenakis denied accepting the traditional relationship of teacher and student and following the rules. He was also disrespectful of those who imposed on him their authority. He often bent rules, stubbornly defending himself, which caused problems in making his music acceptable to many of his teachers.

His father managed to get a false passport of the Dodecanese for Xenakis so he could flee to Italy. Xenakis got into France illegally with the hope of moving finally to the United States, but he stayed in France instead. Xenakis got an engineering job at the office of world-famous French architect Le Corbusier, where many of Xenakis's acquaintances from the Athens

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[11] Nouritza Matossian, *Xenakis* (London: Kahn & Averill, 1986), 25.

[12] Ibid., 28.

Polytechnic were working. It is not that he was interested in architecture, although he found continuation of ancient Greek culture in such ideas as architecture in Paris, but he was just not able to get a job until Le Corbusier hired him in 1948.

He worked at first as an engineer in the project to build an apartment, which would house 1600 people, *L'Unité d'Habitation de Marseille*. Through the experience at Le Corbusier's studio, Xenakis started considering himself as a professional engineer. The puzzling engineering problems and discussions that he and his team were challenged to solve were interesting enough to capture his attention, although he kept composing whenever he found an opportunity. While working on *L'Unité d'Habitation de Marseille*, he became familiar with Le Corbusier's *Modulor*; *Modulor* was the basic unifying idea of this project in terms of scale and dimension, and determined the proportions of the fundamental elements. This proportion that Le Corbusier called *Modulor* shows similarities to the golden section or divine proportion of 5th Century B.C.;[13] it was rather easy for Xenakis to accept this proportional concept because the golden section was used throughout Greek architecture and sculpture. He gradually became an important architectural designer at Le

[13] Golden section will be further explained in the following chapter.

Corbusier's office and kept a very good relationship with Le Corbusier until he had conflicts with him due to the design of the Philips pavilion at the Brussel's World's Fair in 1958.[14]

In the musical society of Paris after the Second World War, the tendency to get away from serialism and neo-classicism prevailed. Composers such as Boulanger and Messiaen were among the most influential forerunners of this movement; they looked for a new method to approach music, to compose new pieces, and to analyze music. In 1949 Xenakis took theory lessons in Honegger's class, which was run by the Ecole Normale; his works were constantly rejected by Honegger for their abundant use of unconventional harmonic progressions and forms. Xenakis, who had little knowledge of music and its history and was remote from Parisian music society, got inestimable advice and hints from Le Corbusier; Le Corbusier's unerring instinct in music lead him to Messiaen, who he believed to be an exception from pompous French composers.[15]

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[14] Le Corbusier claimed that the design of Philips pavilion was his own, but after a long and averse quarrel Le Corbusier declared it was Xenakis's design, in *Le Poème électronique*. Xenakis left Le Corbusier's office in 1960 after this incident. His later architectural projects were mainly used for musical purposes, such as designs of a concert hall and studio for Scherchen's music center, which was abandoned after all, and he adopted architectural principles into his composition.

[15] Matossian, 47.

Unlike other teachers who treated Xenakis's music as a poor exercise, Messiaen advised him not to take conventional theory lessons but to continue to bring in his expert knowledge of architecture and mathematics when he met Messiaen for the first time in 1951;[16] this encouraged Xenakis and changed his attitude toward composition, music, and even his life.

After five years of a good relationship, Le Corbusier offered Xenakis to work with him on the Couvent de la Tourette, with which Le Corbusier himself felt more intimate than with any other projects, and in this project Xenakis contributed quite a good deal of his own design and finally could be truly creative in architectural work, which also might have impacted on his composition. Along with Messiaen's encouragement and Le Corbusier's design offer, Xenakis took advantage of his thorough understanding of architecture, mathematics and science in composing new music. Due to his acceptance of Le Corbusier's *Modulor*, and also because of his Greek heritage, Xenakis showed great interest in the golden mean. This divine proportion was adopted in his music, as mentioned in the preface to *Anastenaria* (Matossian, 1986, p.50) where he insisted that a human body which controls musical

[16] The years that Xenakis had studied with Messiaen differ slightly: in Matossian and Grove dictionary 1951-52, in Varga 1949-50, and in Mario Bois, *Iannis Xenakis* (London: Boosey & Hawkes, 1967) 1950.

durations is found to have divine proportions and therefore musical durations related to the golden sections are the most natural. His new approach to music[17] came to fruition by winning public recognition through festivals and performances.[18] Not only Messiaen and Le Corbusier, but also Françoise, a woman he met in France and married in December 1953, added endless support for his composition.

In 1954, Completion of the design of the Couvent de la Tourette and his orchestral piece *Metastaseis* – two works in totally different fields but based on the same principle of golden section – became a turning point in his life, especially in composition: Le Corbusier credited him in his *Modulor 2*, nicknaming the west façade of the covent "Le Couvent de Xenakis". Chief musicians of contemporary musical circles drew attention to *Metastaseis*,[19] in which he transformed mathematical laws and architectural proportions into compositional procedures that resulted in music. Xenakis started to recognize

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[17] By around 1954, Xenakis's composition reflected the style following Bartok or Sappho's metre as well. (Varga, 28.)

[18] Despite proposals to publish the scores, his music before 1954 remained mostly unpublished.

[19] In these years Xenakis built up the relationships with musicians such as, besides Schaeffer and Scherchen, Pierre Henry, Boulez, and Varèse working in Schaeffer's studio.

himself as a composer as well as a talented architect-engineer.[20] Through Pierre Schaeffer, to whom Messiaen strongly recommended Xenakis and his *Metastaseis*, Xenakis became acquainted with Hermann Scherchen in 1954 and thereafter kept a keen relationship. Xenakis was often invited to Scherchen's studio in Gravesano, Switzerland, for giving lectures at conferences and contributing to the journal, *Gravesaner Blätter*. The articles in this journal later became the sources for Xenakis's book *Music Formelles* (Paris: Editions Richard-Masse) in 1963. The English edition, *Formalized Music: Thought and Mathematics in Music*, was first published by Indiana University Press, Bloomington, in 1971, with chapter IX and appendices I and II added, and he added another four chapters and one appendix for the revised English edition, published in 1992 by Pendragon Press.

Xenakis was very critical of serialism, which prevailed in Europe at that time, for he believed the principle of serialism was too much concerned with pitch.[21] The followers of serialism also saw this as a problem of serial music, a

[20] Matossian,. 75.

[21] Although Xenakis opposed serial music, the fact that mathematical concepts determine the characteristics of musical parameters seems to be departing from serial music. It is not as rigorous as that of total serialist composers in that Xenakis's compositional process was not completely non-human, abstract, or automated but left freedom of making choices after the procedure of mathematical decision. However, his treatment of dynamics in pieces such as

problem that might result in neglecting other values of music such as timbre, duration and intensity. As a result the application of the series was extended to these other parameters. However, Xenakis was not swept along with the general tendency and observed the possible bigger problems caused by using serial structure for some characteristics of music on which serial organization might have little affect, such as timbre. As Xenakis insisted in his article "The Crisis of Serial Music" in *Gravesaner Blätter* in 1955, Music had to be composed in a manner which incorporates elements which change over time, not in geometrical, static ways, for music can not be perceived as an entity that simultaneously represents the whole, but as pieces of a totality, in the flow of time. He believed that since the serial composers attempted to apply the methods which could only be geometrically comprehensible, such as permutations of a given series of notes and their variations of other characteristics, serialism had entered a crisis. Xenakis's response to this crisis was an approach to composition that took into account the continuously evolving nature of the Fibonacci series as well as the probability theory, which opposes strict causality. For all his compositional theories, formulae and graphs were essential and he always consulted with manuals of mathematics

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Evryali and mathematical formulae in composition show the aesthetics of serial music in a sense.

and engineering. However, it seems quite natural that he chose these meticulous ways, for his engineering experience certainly would not have allowed him to be approximate in constructing and designing. Nevertheless, he resisted this influence enough to avoid acceptance of total serialism.

The Probability Theory, which was also used as a mathematical formula to obtain necessary musical components by Xenakis, gave birth to his first piece based on probability, *Pithoprakta* (1956). With this piece he introduced the term "Stochastic"[22] to the music field. As he says, "the laws of the calculus of probabilities entered composition through musical necessity."[23] The Stochastic Law and its adaptations to composition are thoroughly explained in his *Formalized Music*.

While designing the Philips Pavilion for the Brussels World's Fair in 1958, Xenakis also composed a *musique concrète* piece *Concret P.H.* named after the Pavilion; it was performed at the Pavilion with lighting effects which, however, were not so deeply attached to sound. Varèse's *Poème Électronique*, Xenakis's earlier piece *Diamorphoses*, and *Concret P.H.* likewise, used tape

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[22] Further description will be given in the following chapter.

[23] Iannis Xenakis, *Formalized Music* (New York: Pendragon Press, 1992), 8.

manipulation, such as tape transposition, reversal and filtering.[24] After working on *Diamorphoses* in *Groupe de Recherches Musicales* (GRM) and having *Concret P.H.* performed, he became more conscious of the "Abstraction" aesthetics of his music.[25]

From the design of the Couvent de la Tourette to Philips Pavilion and from *Metastaseis* to *Concret P.H.*, the development of Xenakis's compositional ideas found an alliance with his architectural principles. His dismissal of Le Corbusier was largely initiated by the personal betrayal Xenakis had to bear after having been through the quarrel with Le Corbusier on the authority of the design of the Philips Pavilion, but it was also driven by changing circumstances around him. Like any other architects working under an older and famous architect, the work did not fill his pocket satisfactorily, especially since he had a wife and a daughter to support. His attitude at this time was also different from that of some ten years ago. During the interview with Mario Bois in 1967, about his life in Paris in 1947, he recalled:

> Music was something very important to me but not enough for me to live on. . . I finally dropped anchor. . . with Messiaen. . .

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[24] Elliott Antokoletz, *Twentieth-Century Music* (Englewood Cliffs: Prentice Hall, 1992), 459.

[25] Matossian, 122.

> encouraged by him to continue with him. . . That was important. . . But one can't exist on music and a glass of water.[26]

However, the situation had changed and he was fully charged with confidence to try out anything. His music was being performed all over the world and he began to receive commissions. He discovered that it was, in fact, harder to get a contract as an independent architect than to find work as a composer. Working partly as an engineer for a living, his professional knowledge in mathematics, in architecture, and electronic music found firm and nurturing soil for his later pieces such as *Syrmos*, *Analogique*, and *Duel*.

The attempt to formalize his music by breaking down the components and restructuring them into certain stages resulted in separating the time element from other characteristics of music. He set pitch, intensity and duration apart from large-scale temporal relations, so sonic elements could be organized before they are placed into a larger temporal context. The three algebraic stages of composition are the following: outside-time algebra, temporal algebra, and inside-time algebra. Xenakis's thorough explanation on this is given in his *Formalized Music*, with the example of a fragment from Beethoven's piano sonata Op.57. Set theory, which is assumed to be first

[26] Bois, 5.

understood as a basis to comprehend his time sets, specifically Boolean algebra, influenced his piano piece *Herma*, commissioned by and dedicated to Japanese pianist Yuji Takahashi, in 1961; *Herma* will be discussed further in the following chapter.

Xenakis kept developing stochastic law in music and spurred the use of computers as a means of composing. Many of his works received performances and attention from audiences and media. Computers, including those at IBM, accelerated the expansion of stochastic theory into more sophisticated methods, for he could construct and solve more complicated equations of stochastic laws on the computers. Representative works of this period include the *ST* series, *Morsima/Amorsima* and *Atrées*.

His second solo piece, *Nomos Alpha* for cello, however, appeared to break away from his "stochastic-oriented" composition; the theory he developed in this piece is the Theory of Sieves. This theory is essentially a method of creating a series of pitches or rhythms through the application of a formula, which acts as sort of filter, creating symmetrical structures. The theory of sieves helped him to generate computer-synthesized sound as well as scales, rhythms, and structure, showing him possibilities of manipulating the sound signal.

In the hopes of using computers to expand the possibilities of computer generated music, he founded *Equipe de Mathématique et Automatique Musicales* (EMAMu) in 1966, which in 1972 became *Centre d'Etudes de Mathématique et Automatique Musicales* (CEMAMu). With devices such as an electromagnetic drawing table, he could experiment with new possibilities of generating new timbres by composing intuitively without the burden of first programming or notating ahead of time.[27] With this device in particular, he could draw a series of lines in the shape of a tree (leading to the term "arborescence"), which the computer transformed, into musical entities. *Evryali* for solo piano, composed in 1973, is the first example in which the method of arborescence was applied. From 1967, he was a professor at Indiana University at Bloomington, but he resigned in 1972 after the proposal for the creation of a Center for Mathematical and Automated Music was accepted but did not seem to progress further. In 1977, however, *l'Unité Polyagogique Informatique de CEMAMu* (UPIC) was founded in France, where a composer could draw any form of acoustic pressure, amplitude, and pitch and the

[27] Further description of electromagnetic drawing table and its notation system will follow later.

combination of the three would be produced as music. He also taught at the Sorbonne from 1973 to 1989, and there received his doctorate in 1976.[28]

He referred to ancient dramas to compose theatre pieces, which he believed to be the synthetic form of all arts. Its peak is reached with *Polytopes*, where he exploited light sources (laser-beams) in the air, based on the structure of ancient buildings. Later development of stochastic law expanded its realm to the microscopic construction as well, to construct sounds with continuous variations,[29] as shown in one section of *Formalized Music*, "New Proposal in Microcomposition based on Probability Distributions." This, later, was partly used in some sections of piano solo piece, *Mists* (1985).

In the following chapters, mathematical principles such as stochastic law and architectural concepts such as golden section will first be explained in relation to Xenakis's composition. After this theoretical discussion, his first two piano solo pieces, *Herma* and *Evryali*, will be analyzed from the perspective of mathematical and architectural principles. The aspects that the composer declared to be in use in these two pieces, namely set theory and

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[28] The thesis defense recorded was translated in English and published in 1985: *Arts/Sciences: Alloys*, trans. Sharon Kanach (New York: Pendragon Press, 1985).

[29] Xenakis, 246.

arborescence, will be examined. The proportional construction of the works, not mentioned by the composer explicitly, will also be examined.

The fact that Xenakis used a computer to generate his compositions gives rise to a number of issues related to performance. How important is absolute accuracy? Is there room for interpretation in the part of the performer? Does an ideal performance require the use of the computer or electronic device? This matter of performance practice will be also discussed after the mathematics and architecture in his selected piano compositions are studied.

CHAPTER 3
Selected Mathematical and Architectural Methods used in Xenakis's works

Xenakis employed mathematical and architectural principles in his composition, for example, golden section, Boolean algebra, stochastic laws, the arborescence method, and other algebraic concepts, for the construction of pieces and the distribution of musical elements. Therefore, it is important to examine these principles of mathematics and architecture, to better understand Xenakis's composition. Note that the numbers that are used as examples in this chapter do not have musical relevance, unless otherwise stated. In other words, although numbers can be relevant to many parameters of music, such as pitch classes, intervals, and rhythms, because Xenakis's compositional works are multidisciplinary, it is needed to understand numbers as purely mathematical ones in this chapter. It is also clearly stated when musical examples are given with numbers in chapter 3.

3.1 Golden Section

The golden section (GS) has been shown to be in nature, and has served as a formulaic source for architecture, fine art, and music. The golden section is a proportion gained by dividing a line into two parts (short and long) with a point that makes the ratio of the longer part to the shorter part the same as the ratio of the long portion to the whole line (Figure 3-1). There is only one point that makes the golden section; this point is called the Golden Section Point (GSP).[30] The golden section may also be called the Divine Proportion, the Golden Mean, or the Magic Ratio.

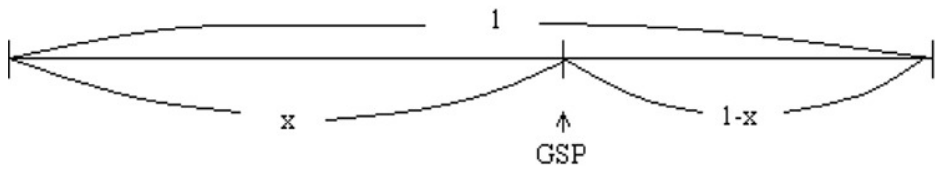

Figure 3-1: Golden Section Point

The ratio:
$x : (1 - x) = 1 : x$, $x = 0.618\ldots$, $1 - x = 0.382\ldots$

The golden section point may also be obtained by construction. Consider a right triangle, ABC (the ninety-degree angle being B), with the ratio AB : BC = 1 : 2. Take D on line AC, the point that makes AD = AB, and take M on BC, which makes CD=CM; Point M is the GSP between B and C (Figure 3-2). In a pentagon, any two diagonals divide each other into golden sections. P is the GSP where AC and BE intersect (Figure 3-3).

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[30] When the longer of the two portions precedes, it is called positive golden section point; when the shorter comes first, negative golden section point.

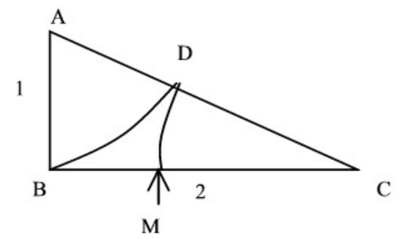

Figure 3-2

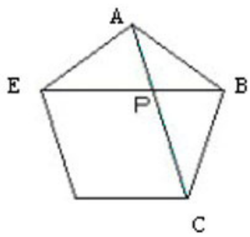

Figure 3-3

The golden section is often found in nature, architecture, and fine arts; examples include the spiral shape of the nautilus shell, the Parthenon, a temple to the goddess Athena (Figure 3-4), and many pictures and sculptures.

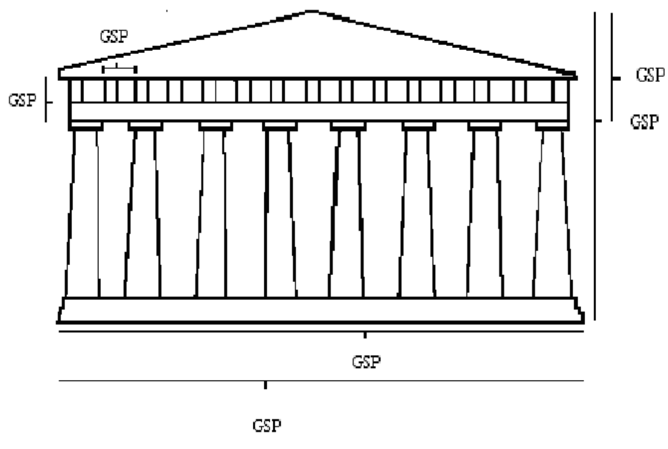

Figure 3-4: GSP of Parthenon

The ratio of the golden section has to do with the Fibonacci Series. The Fibonacci series is a series of numbers in which the sum of the previous two numbers equals the following number. The Fibonacci series is:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 …

As the series goes on (as the numbers get larger), the ratio of each two adjacent numbers approximates to the golden section.

Le Corbusier developed this idea that the proportions in nature, such as shells and trees, could be used as a norm or standard to build houses more effectively after the destruction of the Second World War. Based on golden section, he suggested the proportional value of all the dimensions of a building. He called this proportion the *Modulor*, and used the upright human body with its arm raised as an example as follows:

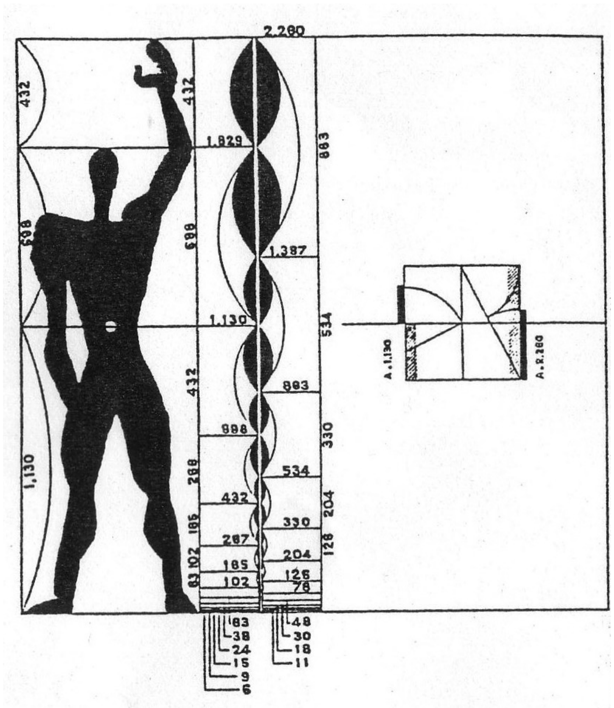

***Figure 3-5: Modulor*[31]**

[31] André Baltensperger, *Iannis Xenakis und die stochastische Musik: Komposition im Spannungsfeld von Architektur und Mathematik* (Bern: Paul Haupt, 1996), 108.

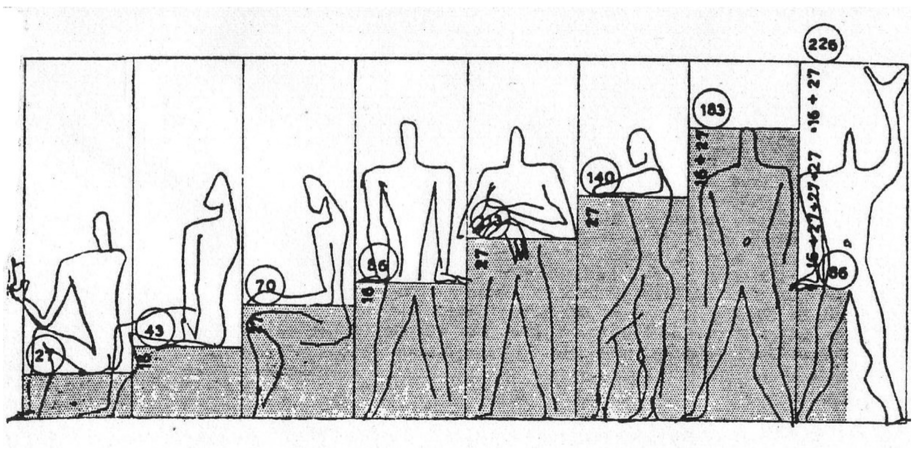

Figure 3-6: Modulor[32]

Xenakis applied these principles of golden section, the Fibonacci series and *Modulor*, to the design of the Couvent de la Tourette. The ratios of the columns on the windowpanes and of the cells follow the numbers of *Modulor* and Fibonacci numbers, and the horizontal metallic joints that connect two vertical columns are placed at golden section points. Figure 3-7 shows the west side façade of the covent.

[32] Ibid., 109.

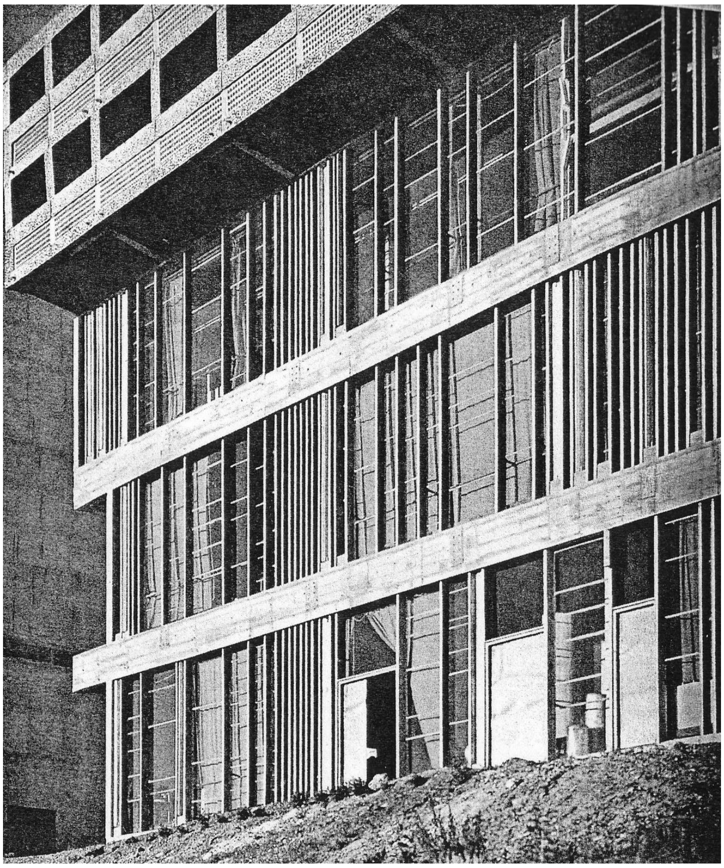

Figure 3-7: The Couvent de la Tourette[33]

He transferred the principle of golden portion of the Couvent de la Tourette into the structure of *Metastaseis*. The sections within each of the four parts of the piece use the numbers of the Fibonacci series to determine duration. The proportions of smaller structures are also planned based on the Fibonacci series and golden section.[34]

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[33] Ibid., 132.

[34] See ibid., 243-257.

3.2 BOOLEAN ALGEBRA/ THEORY OF SETS[35]

Boolean algebra, or the theory of sets, was developed by the English mathematician, George Boole, in the late 1840s. In the theory of sets, basic operations are Union ($\bigcup$), Intersection ($\bigcap$), and complementation or negation (superscript c [^{c}] or line above a symbol), of which logical connectives are *And*, *Or* and *Not*, relatively. If two given sets *A* and *B* have some elements in common, *A* and *B* are said to *intersect*; if two given sets *A* and *B* have no element at all in common, they are *disjunct*; if every element of one set *A* is also an element of the other set *B* and every element of *B* is an element *A*, they are *equal* [$A = B$]; if every element of set *B* is found in set *A* but not every element of *A* is found in set *B*, *B* is included in *A*, *B* is a subset of *A* [$B \subset A$].

The Venn diagram, developed by the logician John Venn (1834-1923),[36] is generally used as a geographical means to visualize the relations of sets in the set theory. The referential or universal set is usually drawn in a rectangle and the individual sets in circles. The region that needs to be shown is generally shaded (Table 3-1).

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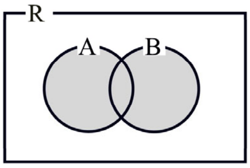
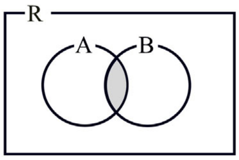
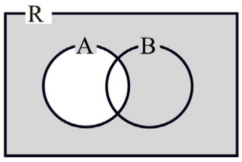
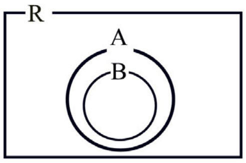
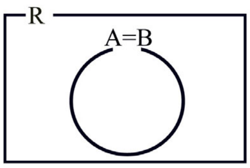
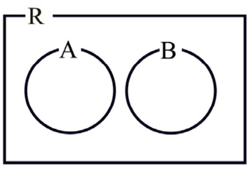
[35] Adopted from Spencer, Miller, Xenakis, Whitesitt, and Bartsch.

[36] Charles Miller, *Mathematical Ideas* (Glenview, Illinios: Scott, Foresman, 1973), 59.

Table 3-1

Set Theory	Boolean algebra	Venn Diagram	Term(s)	Definition
$A \bigcup B$	$A + B$		Union, Logical sum	All the elements that are in A or B. (sum of A and B)
$A \bigcap B$	$A \cdot B$, AB		Intersection, Logical product	The elements that are common to both A and B
A^c	$\bar{A}$		Complement, Negation	All elements that are NOT in A
$B \subset A$	$B \subset A$		Subset	B is included in A
$A = B$	$A = B$		Equal	All elements in A are same as in B
$A \bigcap B = \emptyset$	$A \neq B$		Disjoint	No common element between A and B

Given that the referential or universal set R is:

$R = \{x|\ 0 < x \leq 20$, x is a natural number$\}$

Let there be two sets A and B, of which elements are from the referential (or universal set) R:

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

The union of A and B consists of all the elements of A and all of B; therefore:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

The intersection, which comprises of the elements that belong to both A and B at the same time, thus will be:

$A \cap B = \{6, 7, 8, 9, 10\}$

The complement of set A is the set of all the elements that are in the referential R but not in the set A, which means:

$A^c = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

Likewise, B^c is:

$B^c = \{1, 2, 3, 4, 5, 16, 17, 18, 19, 20\}$

In case the referential set R was not given, it becomes problematic to determine what the complement of set A will be, since anything not in the set A

may be in the complement set; therefore, it is necessary to clarify the universal set. Figure 3-8 shows the Venn diagram of the given example.

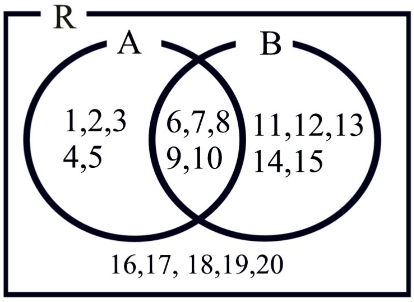


Figure 3-8

Boolean algebra is the main mathematical principle that is used in composition of *Herma*, Xenakis's first piano piece composed in 1961. The structure and pitch constructions of *Herma* are determined on the mathematical set theory, Boolean algebra. Given the referential set *R* is 88 keys of the piano and the notes of sets *A*, *B* and *C* are:

A = {A4, B4, C4, D4, E4, F4, G4, A5, C5, E5, G#5}

B = {C4, D4, E4, F4, G4, G#4, A5, B5}

C = {E3, G#3, C4, E4, G#4, C5}

The operations given below[37] result in:

AB = {C4, D4, E4, F4, G4, A5}

AC = {C4, E4, C5}

[37] Examples of operations are taken from *Herma*.

BC = {C4, E4, G#4}

ABC = {C4, E4}

$A\overline{B}\overline{C}$ = {A4, B4, E5, G#5}

$(AB+\overline{A}\overline{B})C$ = {E3, G#3}

$\overline{(AB+\overline{A}\overline{B})}\overline{C}$ = {A4, B4, B5, E5, G#5}

The Boolean operations share certain principles with algebraic laws (Table 3-2).[38]

Table 3-2

(0 is null set; 1 is referential)

Commutative Laws:	1) $A+B=B+A$ [39]
	2) $A \cdot B = B \cdot A$
Associative Laws:	1) $A+(B+C)=(A+B)+C$
	2) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Distributive Laws:	1) $A \cdot (B+C) = (A \cdot B)+(A \cdot C)$

[38] Some parallel algebraic properties of the Boolean operations expanded Xenakis's theory from set to group structure, as in mathematics. Elements of each set are applicable to the following four properties, and therefore a set has a group structure:

closed binary rule; pairs of elements of a set are to make a connection to produce the third one

associative property; either adding the second one to the first one and then the third one, or adding the third to the second and then the first, would produce the same fourth one. Parallel to associative law.

identity/neutral element; there is an element that makes the operation result unchanged. Parallel to laws of identity.

an inverse; every element of a set has its inverse, which makes the operation undone. Parallel to laws of complementation. ex. –1 + 1 = 0

[39] Therefore, $AB+\overline{A}\overline{B}$ at measure 143 and $\overline{A}\overline{B}+AB$ at measure 145 of *Herma* result in the same set of notes.

2) $A+(B\cdot C)=(A+B)\cdot(A+C)$

Laws of Identity:	1) $A+0=A$
[Operations with 0 and 1]	2) $A+1=1$
	3) $A\cdot 0=0$
	4) $A\cdot 1=A$
	5) $\overline{0}=1$
	6) $\overline{1}=0$
Laws of Tautology:	1) $A+A=A$
	2) $A\cdot A=A$
Laws of Complementation:	1) $A+\overline{A}=1$
	2) $A\cdot\overline{A}=0$
De Morgan's Laws:	1) $(\overline{A+B})=\overline{A}\cdot\overline{B}$
	2) $(\overline{A\cdot B})=\overline{A}+\overline{B}$
Laws of Absorption:	1) $A+(A\cdot B)=A$
	2) $A+(\overline{A}\cdot B)=A+B$
	3) $A\cdot(A+B)=A$
	4) $A\cdot(\overline{A}+B)=A\cdot B$
Law of Involution:	1) $\overline{\overline{A}}=A$
(Law of Double Complementation)	

Truth Table is used to show the Boolean algebraic properties of equivalence. Given that variables x and y are used, each can be examined and compared with the other. The four possibilities are: x and y both false; x true y false; x false y true; and x and y both true. Only when the values are true for both cases, the conjunctive value is true. 0 is to be a proposition that is always false, and 1 to be a proposition that is always true (Table 3-3).

Table 3-3

		x	
		0	1
y	0	0	0
	1	0	1

Eight possible combinations of three classes A, B, C are found by logic operation as follows:

$$A \cdot B \cdot C, A \cdot B \cdot \overline{C}, A \cdot \overline{B} \cdot C, A \cdot \overline{B} \cdot \overline{C}, \overline{A} \cdot B \cdot C, \overline{A} \cdot B \cdot \overline{C}, \overline{A} \cdot \overline{B} \cdot C, \overline{A} \cdot \overline{B} \cdot \overline{C}.$$ [40]

Another simplification method is given in the form of the Karnau Map (Table 3-4) and the Truth Table.

Table 3-4: Karnau Map

		B			
		0	0	1	1
A	0	$\overline{A} \cdot \overline{B} \cdot \overline{C}$	$\overline{A} \cdot \overline{B} \cdot C$	$\overline{A} \cdot B \cdot C$	$\overline{A} \cdot B \cdot \overline{C}$
	1	$A \cdot \overline{B} \cdot \overline{C}$	$A \cdot \overline{B} \cdot C$	$A \cdot B \cdot C$	$A \cdot B \cdot \overline{C}$
		0	1	1	0
		C			

In this Karnau Map map the upper row represents the locations of $\overline{A}$, and the lower row those of A. First two columns show the settings of $\overline{B}$ with

[40] Every Boolean expression or function of the three classes *A*, *B,* and *C* can be expressed in *disjunctive canonic*: $\sum_{i=1}^{8} \sigma_i k_i$

where $\sigma_i = 0$; 1 and

$k_i = A \cdot B \cdot C, A \cdot B \cdot \overline{C}, A \cdot \overline{B} \cdot C, A \cdot \overline{B} \cdot \overline{C}, \overline{A} \cdot B \cdot C, \overline{A} \cdot B \cdot \overline{C}, \overline{A} \cdot \overline{B} \cdot C, \overline{A} \cdot \overline{B} \cdot \overline{C}$ (Xenakis, 173).

the next two columns those of *B*. Middle columns represent *C*, whereas the outer columns $\overline{C}$.

Such equation as:

(1) $F = A \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C$,[41]

where each of the three sets intersects with the other two, can be found in Karnau Map (shaded cells in Table 3-5):

Table 3-5

$\overline{A} \cdot \overline{B} \cdot \overline{C}$	$\overline{A} \cdot \overline{B} \cdot C$	$\overline{A} \cdot B \cdot C$	$\overline{A} \cdot B \cdot \overline{C}$
$A \cdot \overline{B} \cdot \overline{C}$	$A \cdot \overline{B} \cdot C$	$A \cdot B \cdot C$	$A \cdot B \cdot \overline{C}$

and also be drawn in the Venn diagram (Figure 3-9).

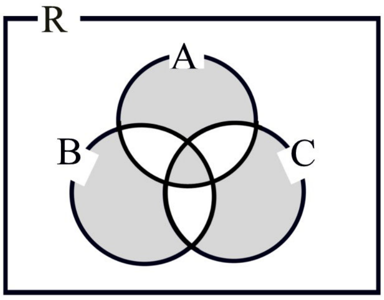


Figure 3-9

The above function F can also be expressed as follows:[42]

[41] Equations (1) and (2) are presented on the final page of *Herma*.

[42] Ibid.

$$(2)\ F = (A \cdot B + \overline{A} \cdot \overline{B}) \cdot C + (\overline{A \cdot B + \overline{A} \cdot \overline{B}}) \cdot \overline{C}$$

The flow charts of both equations (1) and (2) are given in Chapter 4, in relation to *Herma*.

3.3 STOCHASTIC LAWS

A random experiment has an unpredictable outcome. Examples include such things as tossing a coin, playing the lottery, or rolling dice. The probability of a certain outcome ranges from zero to one; zero when an expected result does not happen at all, and one when the performance brings about the expected result.[43]

Stochastic Theory shows the probability that as the number of performed experiments becomes larger, the result tends to become determinate. The variables to be executed by stochastic law should be randomly chosen. This theory is being used in natural sciences, such as agriculture and clinical methodology, as well as social sciences including census. When there are 100 patients who have their blood pressure checked, some would show high blood pressure, some others low pressure. However, the majority of the result will be within the normal blood pressure range (Gaussian distribution). This example can be drawn as follows on a graph (Figure 3-10):

---

[43] Given Ω is the sample space, the set of all possible outcomes, random variable's distribution function *m* is defined by Ω as follows:

$$\sum_{x_i \in \Omega} m(x_i) = 1$$

The possibilities that elements of set A would happen is:

$$P(A) = \sum_{x \in A} m(x)$$

$$0 \leq P(A) \leq 1, P(\emptyset) = 0, P(\Omega) = 1 \ (\emptyset : \text{null set})$$

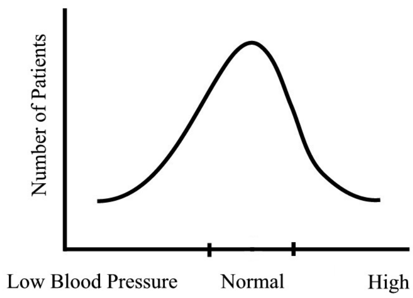


Figure 3-10

In music, note durations and their successions can be determined by a stochastic formula. This formula is used for such case as throwing a dart on a spinning wheel, where the given experiment set is continuous, and shows the probability that a segment x of a continuous set A happens.

$$P(x) = \delta \cdot e^{-\delta x} dx$$

δ is the density, the total number of sounds during the given measure or given time unit (eg. sound events/ second). The larger a density becomes, the more sound events occur. The exemplary graph of this function $P(x)$ for given densities δ_1= 2.5 and δ_2= 4.5 is as follows:

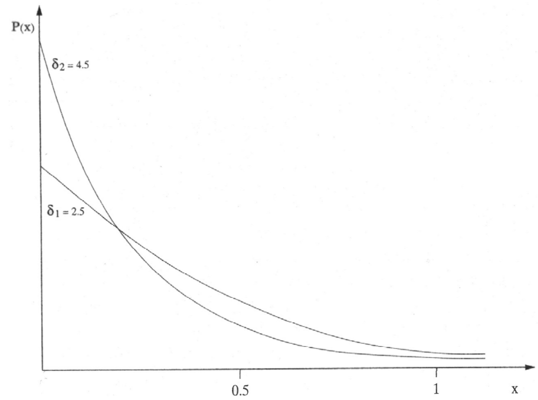


Figure 3-11: Probability graph of x, with given densities[44]

The stochastic law determines the clouds of sound as well in Xenakis' music. Groups of notes, rather than individual pitches, are perceived by the listener, and these sounds form a shape, which Xenakis called a cloud. In this way, the whole shape is more significant and meaningful than detailed placement of notes. A cloud, or a group of sound events, is determined by two factors: the density and intervallic content. The average density of a piece depends upon the number of actual notes playable in a given time period. The nature of the instrument, the number of instrumentalists, and technical difficulties are considered in determining the density.[45] The intervals between the pitches of a cloud are defined by another continuous probability law, which

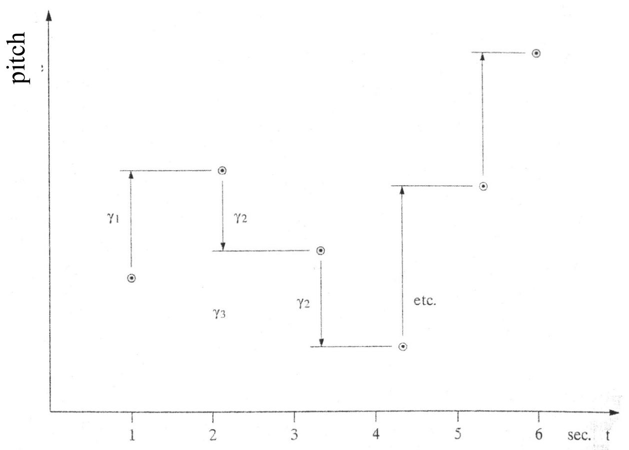
[44] Baltensperger, 447.

[45] Xenakis, 136.

gives the probability that, within a segment of length a, a segment of a length within γ and $\gamma + d\gamma$ will occur:

$$P(\gamma) = \frac{2}{a}(1 - \frac{\gamma}{a})d\gamma$$

The intervals between the points decide the distribution of clouds, as shown in Figure 3-12:[46]



γ : interval, f: frequency, t: time (second)

Figure 3-12: Cloud (tone points)

The density of any given point within the set of sound events is the number of sound events within a given time unit (eg. sound events/second), and a cloud has a density δ chosen by the composer. The possibility $P(k)$ that the

[46] Baltensperger, 448.

event occurs k times per unit of measurement, which is time in Xenakis' composition, follows the Poisson distribution:[47]

$$P_k = \frac{\delta^k}{K!} e^{-\delta}$$

The Poisson distribution is one of most frequently used formulae in Xenakis' pieces that are stochastically composed, such as *Achorripsis*.[48] The mean density δ could be chosen for convenience,[49] and the probabilities that certain events occur within a given time frame or within given measures can be calculated based on the mean density and the number of event, k. Once the probabilities are obtained by the Poisson's formula with the given density, the distribution of events is determined accordingly.[50]

Stochastic theory is used in composition of *Herma*, for the time interval between the occurrences of sets, according to the composer himself, yet the specific application is not described. However, the densities are set by the composer and the sound events occur relatively accordingly.

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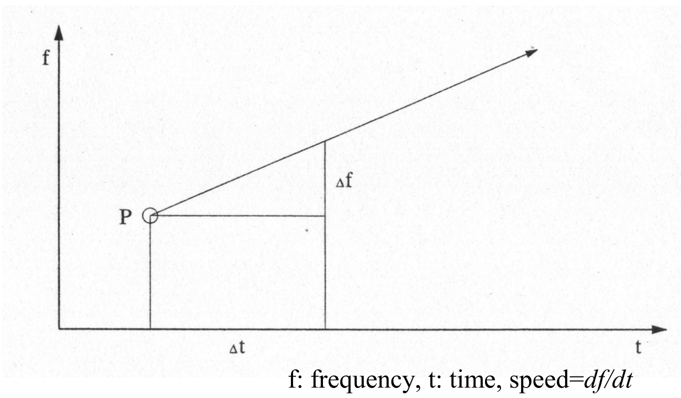
[47] Distribution of clouds also uses Poisson's law.

[48] The close analysis of *Achorripsis* based on stochastic law is given in Xenakis, 26 – 38.

[49] Ibid.

[50] See ibid. for detailed explanation.

The probability laws characterize glissandi as well. The acoustic characteristic of a glissando assimilates the speed (*df/dt*) of a continuous movement (Figure 3-13) [51] and the distribution of speed follows another law of probability (Gaussian law).[52]



f: frequency, t: time, speed=*df/dt*

Figure 3-13: Acoustic characteristic of a glissando, Speed

Such aspects of a glissando as the duration, the dynamics and the moment of occurrence are determined by continuous random probability. As shown in the example of *Achoripsis*, the dynamics and their combinations can also be ruled by stochastic principles.[53]

---

[51] Baltensperger, 451.

[52] Ibid.

[53] See Xenakis, 32 – 38, 134 – 143, for more details.

The examples, which show the use of stochastic theory in determining the glissandi, include *Metastaseis* and *Achorripsis*. The composition of *Metastaseis*, as indicated in Figure 3-14[54] and Figure 3-15,[55] formulated the architecture of the Philips Pavilion. The stochastic law formed not only music but also his architectural projects, which affected each other (Figure 3-16[56] and Figure 3-17[57]).

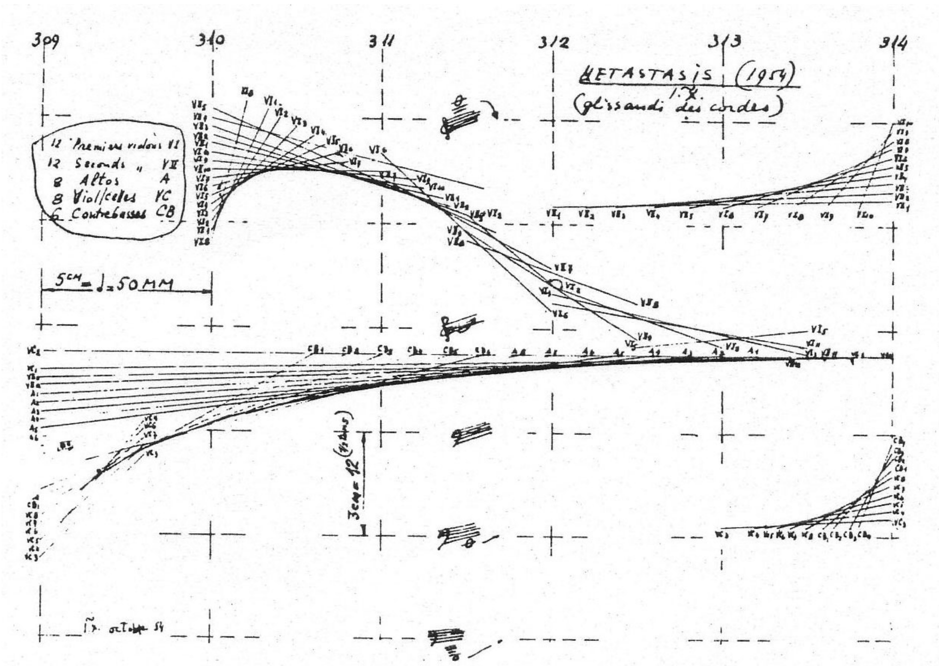


Figure 3-14: Glissandi of Metastaseis

[54] Xenakis, 3.

[55] Ibid., 10.

[56] Philips Technical Review, Vol. 20, No. 1 (1958/59), p.1.

[57] Philips Technical Review, Vol. 20, No. 1 (1985/59), p.35.

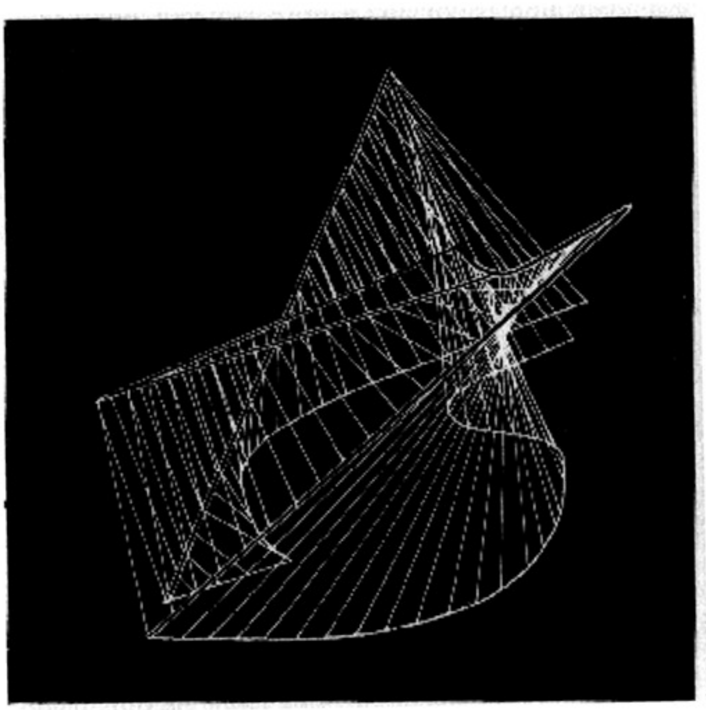


Figure 3-15: First Model of Philips Pavilion

The influence of stochastic principle that began with music (*Metastaseis*) and then used in architecture (the Philips Pavilion) was transferred back to music, *Concret P.H.*, which was composed for played in the pavilion. With the composition of *Concret P.H.*, Xenakis analyzed the stages of composition into eight steps so that he could adapt these for later compositions on computer programmes[58]:

[58] Ibid., 22.

1. Initial conceptions;

2. Definition of the sonic entities: material;

3. Definition of the transformations: macrocomposition, general choice of logical framework;

4. Microcomposition: detailed fixing of the functional or stochastic relations of the elements of 2.;

5. Sequential programming of 3. and 4.: pattern of entire work;

6. Implementation of calculations: feedback and modifications of the sequential program;

7. Final symbolic result of the programming: notation;

8. Sonic realization of the program: performance.

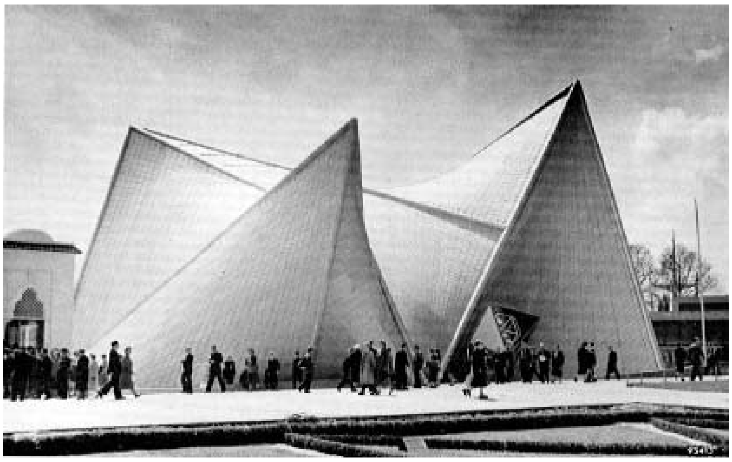


Figure 3-16: General View of Philips Pavilion

Figure 3-17: Exterior of entrance of Philips Pavilion

3.4 Sieves

One of the methods Xenakis used in regards to pitch construction is a sieve, which might be also considered parallel to constructing a scale. A sieve is a theory that allows symmetries to be built from a given series of events. Any ordered set can be constructed through a sieve, including scales.[59] Let the representing function x_y, where x is modulus, the unit distance, and y is indice, the starting point. Therefore, the sets with integers are:

3_0= {0, 3, 6, 9, 12, 15, …}

4_0= {0, 4, 8, 12, 16, …}

We obtain Union U = {0, 3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21, 24, 27, 28, …} from 3_0 U 4_0 and Intersection I = {0, 12, 24, 36, … } from$3_0 \cap 4_0$. The smallest common multiple of 3 and 4 is 12; therefore I can also be given as 12_0.

Given that note C is the zero and integer 1 designates one semitone, let the pitches apply to the functions above, 3_0 and $4_{0.}$

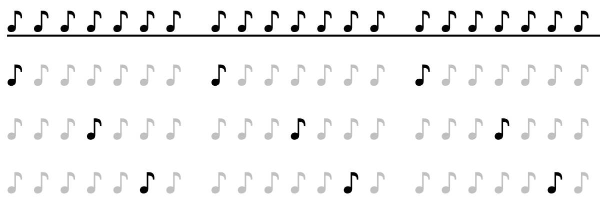
3_0= {C, D#, F#, A, C, …}

4_0= {C, E, G#, C, E, …}

Therefore, set U is {C, D#, E, F#, G#, A, C, D#, E, F#, …} and set I is a set of Cs of every octave, which is 12_0.

[59] Xenakis, 268.

Duration and time intervals can also be achieved by the sieve theory. Given 7_0= {0, 7, 14, 21, …}, 7_3= {3, 10, 17, 24, …}, and 7_5= {5, 12, 19, 26, …}, 7_0 U 7_3 U 7_5= {0, 3, 5, 7, 10, 12, …}; when 1 represents one eighth note, 7_0 U 7_3 U 7_5=



therefore,

Having more than two sets that are generated by the smallest common multiple and the largest common denominator of the moduli of given first two sets will make the algorithm vary; denser structures will be obtained by union, rarified structure by intersection, and more complex structure by simultaneous use of both operations. Any other elements of sonic characteristics, such as loudness, can be given numerical values (positive integers) and they can be determined in the same way as above.

3.5 ARBORESCENCE

To create a compositional method related to causality, repetition and consequent variation,[60] and to keep the music from losing continuity, Xenakis brought up the idea of arborescence. He opposed contemporary electronic music, where composers let an oscillator choose similar sine waves and modified them through filtering, splicing, transposing and so on; he found the sound produced as a result to be dull and unsatisfying. Thus he programmed a graphic electromagnetic system (electromagnetic table) at *Centre d'Etudes de Mathématique et Automatique Musicale*s (CEMAMu), so that a composer creates a graph with pressure and time axes, for example. In this way, he determines "what we actually hear," at the first stage. The electromagnetic table where one can draw directly with an electromagnetic pencil could interpret the randomly drawn lines into certain curves on either pitch versus time [melody], pressure versus time [intensity], or frequency graph [timbre], and could generate sound based on the curve signal.

A human ear perceives the outcome of atmospheric pressure that occurs in time. This pressure changes in time, creating sinuous curves. A pure tone has a single sine wave. Noise, on the other hand, shows a very irregular

[60] Varga, 88.

pattern; pitches, musical tones, show regular patterns, which result from the combination of different sine waves.

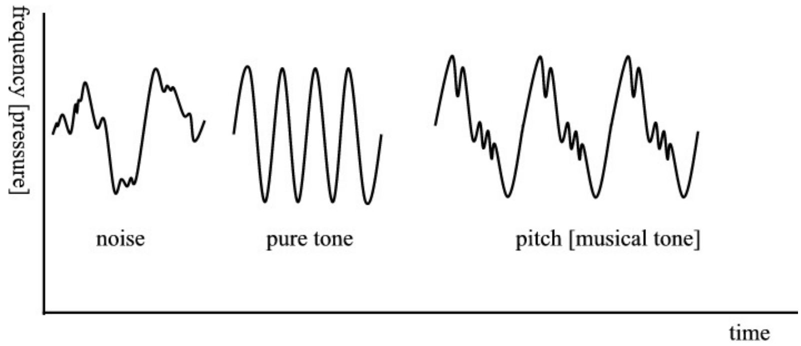


Figure 3-18

Any given curvy line may reproduce itself so it becomes a bush or a tree shape [arborescence], and the tree or bush shape is placed on pitch versus time space. The initial tree or bush might be transformed, through rotation, inversion, retrograde, or their combination, in music; transformed melodies can also be treated as a group. There are more possibilities in the method of arborescences, for the rotations can be made at any angle.[61]

The three necessary elements of a sound that need to be drawn are:

1. Timbre: a form representing the acoustic pressure versus time;
2. Envelope: a form representing the amplitude versus time;
3. Curve of pitch/time: a form representing the pitch versus time.[62]

---

[61] Ibid., 89.

[62] Matossian, 241.

When these three aspects are combined, traditional music notation of the curvy lines become possible.

3.6 OUTSIDE–TIME ALGEBRA, TEMPORAL ALGEBRA, AND INSIDE–TIME ALGEBRA

Pitch, intensity, and duration are given regardless of time relation, and therefore can be set by algebra before temporal elements are given. Given three elements, pitch, intensity, and duration, it is possible to combine them to embody certain musical notation, using three-dimensional vector space graphs. This is algebra outside-time. The element of time, or the pacing of events within a composition, is set independently of the sonic element, only with consideration for the moment of attack in the flow of time. This is called temporal algebra. Once these two sets are given, it becomes possible to put them together into sonic realization in time. This is in-time algebra.

Outside-time relationships are established by assigning values of pitch, intensity, and time intervals onto a multi-dimensional vector space. According to Xenakis:

> Sets H (melodic intervals), G (intensity intervals), U (time intervals), and T (intervals of time separating the sonic events, and independent of them) are totally ordered. . . . Let $\overline{X}$ be a sequence of three numbers x_1, x_2, x_3, corresponding to the elements of the sets H, G, U, respectively,

and arranged in a certain order: $\overline{X} = (x_1, x_2, x_3)$. This sequence is a vector and x_1, x_2, x_3 are its components.[63]

Vector $\overline{X}$ of $\vec{H}$, $\vec{G}$ and $\vec{U}$, with their elements $\vec{h}$, $\vec{g}$ and $\vec{u}$, can be drawn as follows (Figure 3-19):

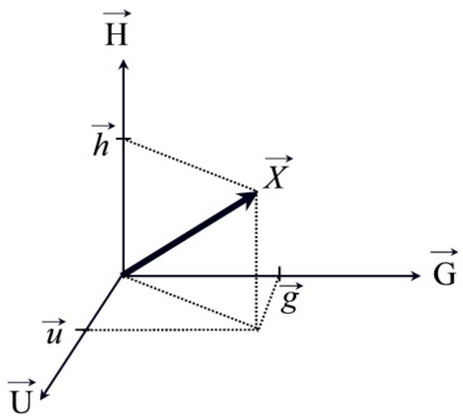


Figure 3-19: Vector Graph

For its sonic realization, let $1\vec{h} =$ 1 semitone, $1\vec{g} =$ 10 decibels, and $1\vec{u} =$ 1 second, where the original points O of $\vec{h}$, $\vec{g}$ and $\vec{u}$ are C4, 50 db, and 10 seconds, respectively. Such vectors as $\vec{X} = 5\vec{h} - 3\vec{g} + 5\vec{u}$, when 1 second is MM=45, can be notated as follows (Figure 3-20):[64]

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[63] Ibid., 161.

[64] Example has been taken from ibid., 163.

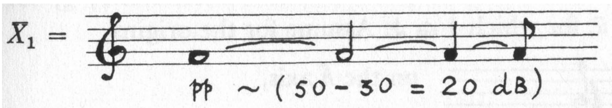

Figure 3-20

After the outside-time algebra is executed and the sonic events have been chosen, temporal succession, without consideration of sonic events, is created. This decides how the entries of sonic events should occur in consideration of time. In other words, it determines if 'A' should occur before 'B,' or vice versa.

When outside-time algebra and temporal algebra are operated independently, the time parameter set T ($\{t | t \in T\}$) can be associated with sonic elements in vector space. Therefore, $\overrightarrow{X} = H(t)\vec{h} + G(t)\vec{g} + \mathrm{U}(t)\vec{u}$, where t becomes another factor of vector, can be given. The example is given by Xenakis (Figure 3-21):[65]

[65] Ibid., 166.

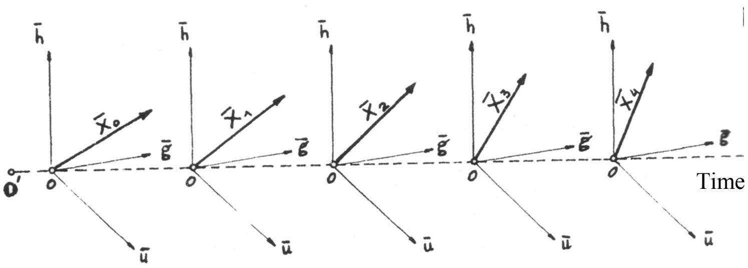

Figure 3-21

Its sonic realization in the first movement of Beethoven sonata Op. 57 is as follows (Figure 3-22, upper staff):

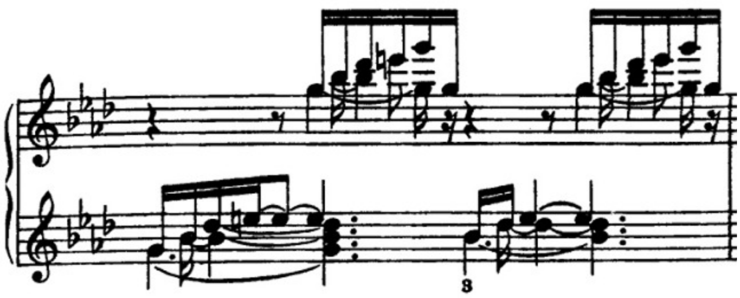

Figure 3-22: m. 124, Beethoven's Sonata Op. 57, First Movement

CHAPTER 4

Mathematical and Architectural aspects featured in *Herma* and *Evryali*

4.1 *HERMA*

Xenakis's first piano solo piece *Herma*, "bond" or "foundation,"[66] was commissioned by and dedicated to the Japanese pianist Yuji Takahashi, with whom Xenakis started a keen relationship when Xenakis visited Japan in 1961 for the International Congress of East and West. Composed in 1961, the compositional source came from mathematics, especially from Boolean algebra. Three sets made up of pitches are set up initially and they are subjected to follow certain mathematical functions that are given by the composer.

4.1.1 Outside-time, temporal, and inside-time structure

In Xenakis' piano solo piece *Herma*, Boolean algebra was used to set up outside-time materials and inside-time elements. However, the temporal element set (set T) of *Herma* was not derived by way of set theory for this theory is totally non-temporal. Although Xenakis tried to arbitrate this problem with a unique explanation that the function of time in *Herma* is that kind of

[66] Matossian, 151.

absolute, true and mathematical time, which flows equably without relation to anything external,[67] in accordance with Einstein's view, his attempt does not seem to accomplish much. He also mentioned briefly "a stochastic correspondence between the pitch components and their moments of occurrence in set *T*, which themselves follow a stochastic law,"[68] but no further description was given. A formula derived from the principles of continuous probability provides possibilities of all length when the mean number of points to be placed and the density are given.

$$P_x = \delta e^{-\delta x} dx$$

(δ is the linear density of points)

The linear densities for sets are given, as shown in figure 12.

The elements of the sets, notes, are unfolded at random and amorphously, without any internal structure, so that the notes do not follow any strict law of melody and the sets are independent from each other.[69]

Three sets of notes, of which referential set is 88 keys on the piano, are as follow:

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[67] Ibid., 154.

[68] Xenakis, 175.

[69] Varga, 84-5.

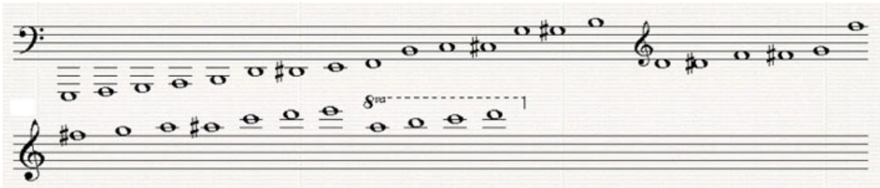

Figure 4-1: Set A

Figure 4-2: Set B

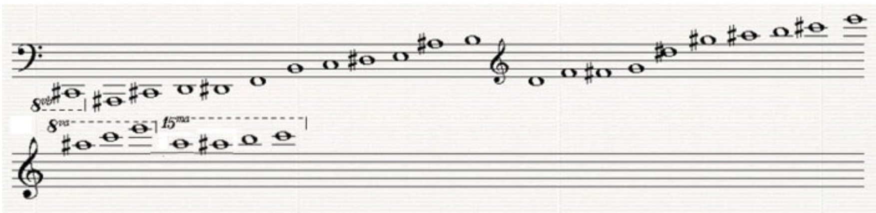

Figure 4-3: Set C

Pitches of three sets in *Herma* can be drawn as Boolean classes as follows (Figure 4-4):

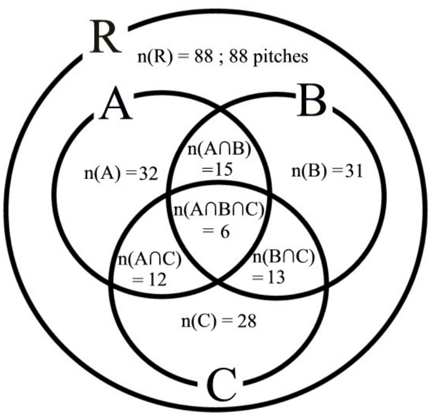

Figure 4-4

The sets of pitch for *Herma* follow the equation below as given on the last page of the score:

$$[F = A \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C = (A \cdot B + \overline{A} \cdot \overline{B}) \cdot C + \overline{(A \cdot B + \overline{A} \cdot \overline{B})} \cdot \overline{C}]$$

: first part – equation (1), second part – equation (2)

The flow chart of equations (1) and (2) are shown below (Figure 4-5, after Xenakis).[70]

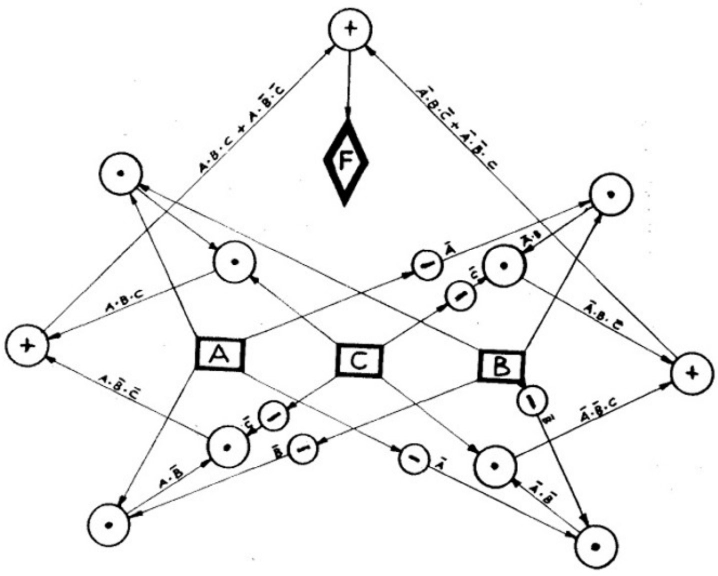

Figure 4-5: Flowchart for* Herma *#1

Equations (1) and (2) on two parallel planes can also be shown as Figure 4-6.[71]

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[70] Xenakis, 173.

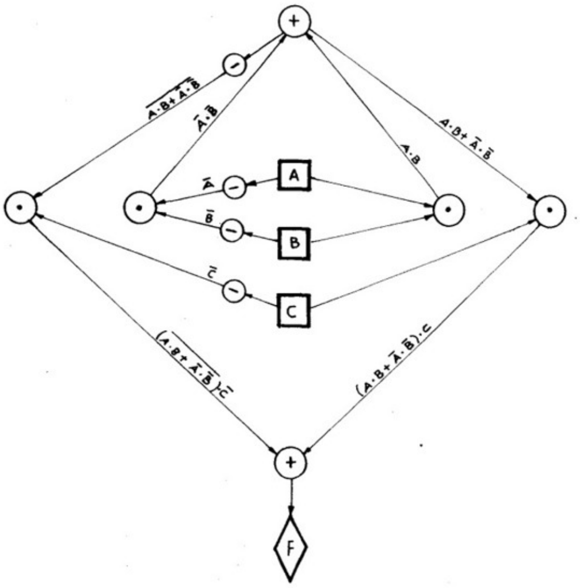
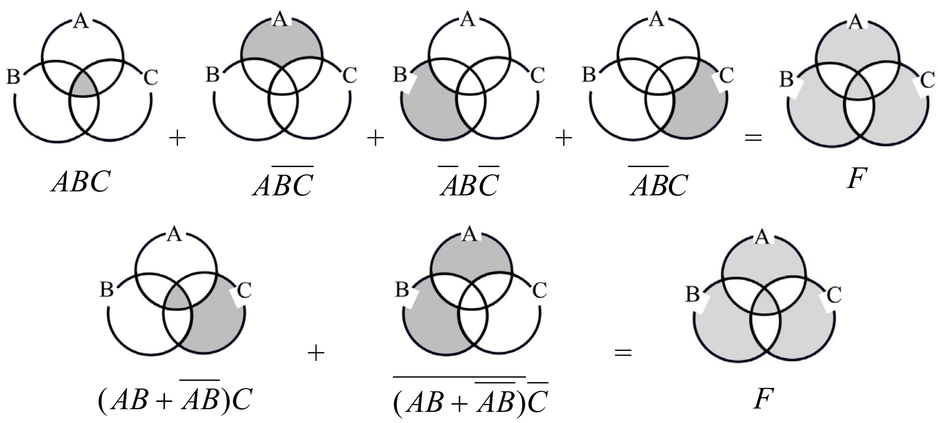


Figure 4-6: Flowchart of* Herma *#2

Two operations of equations (1) and (2) are drawn as below using Venn diagram:

Table 4-1



[71] Ibid.

4.1.2 Formal Structure of *Herma*

The basic formal plan of *Herma* is based on distribution of the sets on two planes as shown in Figure 4-7 (after Xenakis, p.176).[72] Each plane represents equation (1) and (2) respectively and is subdivided into two distinctive dynamic levels: *f* and *fff*, *ff* and *ppp* correspondingly. The sets function as distinctive musical sections. Sets *R*, *A*, *B*, and *C,* and their complements that are presented successively at the beginning of the piece, are not shown in either plane. Sets *A*, *B*, and *C* are assigned to two contrasting dynamics, *pp/ff*, *f/pp*, and *ppp/ff* respectively, and their complements to *ff*. Dynamic levels for other sets are given by Xenakis.

[72] There appears to be a typographical error in the chart in *Formalized Music* p.176. $\overline{(AB+\overline{AB})}C$ that occurs twice in both *ff* and *ppp* levels at the end of plane (2) should be corrected to $\overline{(AB+\overline{AB})}\overline{C}$ (*C* to *C* negation).

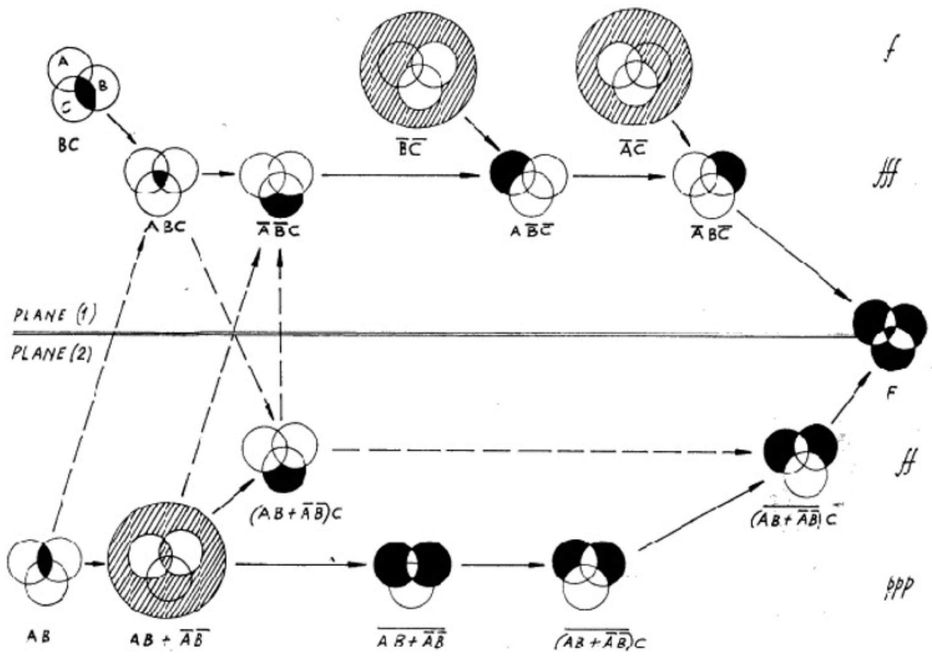


***Figure 4-7: Two planes of* Herma**

Figure 4-8 shows the precise plan of the construction of *Herma*.[73]

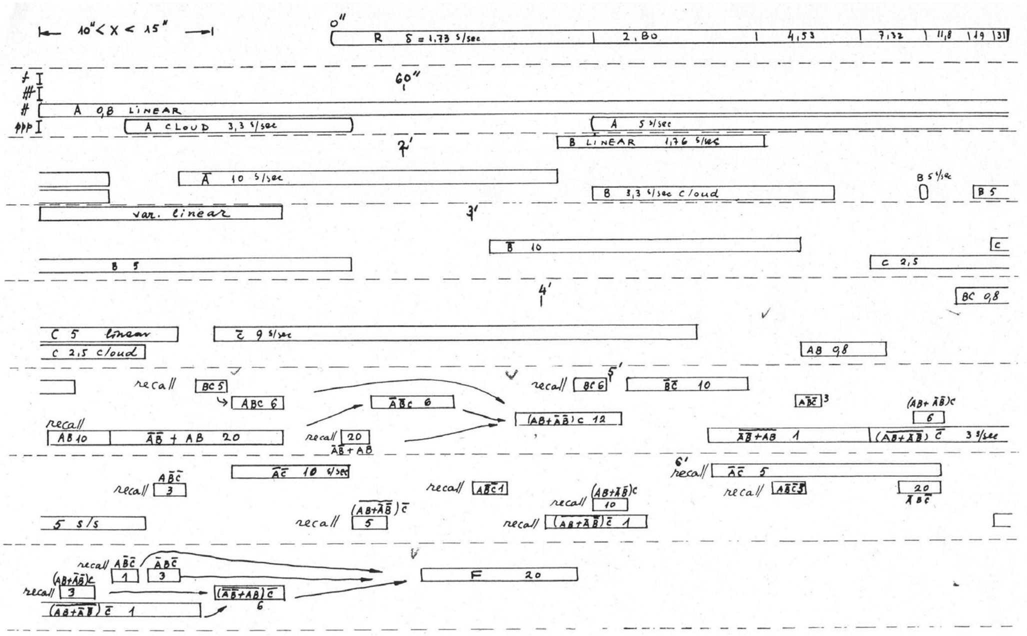


***Figure 4-8: Construction of* Herma**

Table 4-2 shows the sets unfolding, with tempo changes, the plane they belong to from Figure 4-7, dynamics, duration of sets and accumulated time (given in seconds). Measure numbers and meters are given as well for reference, although Xenakis himself noted at the beginning of music that measures and meter signs do not hold any subdivision or accentuation but only serve as divisions in time. As noted by the composer, union operation of sets is shown by "+" and intersection by juxtaposition of letters. Remarks are shaded

[73] Ibid., 177.

when two different operations are juxtaposed with different density and dynamics. The information in this table is also used to calculate the proportions of durations and densities.

Table 4-2

Measure	1	30	32	39	46	60	62	73	74	80	81	82	84
Set	*R*	*A* linéaire	+*A* nuage	*A* linéaire	+*A* nuage	Rest	$\overline{A}$	*B* linéaire	+*B* nuage	Rest	*B* linéaire	Rest	*B* nuage
Meter		12/8											
Tempo MM	104 ->120	180											
Plane													
Dynamics	*ppp* to *fff*	*ff*	*pp*	*ff*	*pp*		*ff*	*f*	*pp*		*f*		*pp*
Density		0.8	3.3	0.8	5		10	1.8	3.3		5		5
Duration (second)	33	4	14	14	28	4	22	2	14	2	2	4	2
Accumulated Time (second)	33	37	51	65	93	97	119	121	135	137	139	143	145
Remark													

Table 4-2 Continued

Measure	Set	Meter	Tempo MM	Plane	Dynamics	Density	Duration (second)	Accumulated Time (second)	Remark
85	+*B* (var) linéaire				*f*	5	14	159	
$91^1/_2$	*B* nuage				*pp*	5	4	163	
94	Rest						8	171	
98	$\overline{B}$ nuage				*ff*	10	18	189	
107	Rest						4	193	
109	*C* nuage				*ppp*	2.5	8	201	
$112^1/_2$	+*C* linéaire				*ff*	5	10	211	
117	Rest						2	213	
118	$\overline{C}$				*ff*	9	28	241	
132	Rest						6	247	
135	*AB*			2	*ppp*	0.8	6	253	
138	Rest						4	257	
140	*BC*			1	*f*	0.85	2	259	

Table 4-2 Continued

Measure	141	143	$145^1/_2$	$146^1/_2$	149	150	151	153	155	$156^1/_2$	$157^1/_2$
Set	AB rappel	$AB+\overline{AB}$	$+BC$ rappel	$+ABC$	Rest	$AB+\overline{AB}$ rappel	$\overline{AB}C$	Rest	$(AB+\overline{AB})C$	$+BC$ rappel	$(AB+\overline{AB})C$ toujours
Meter											
Tempo MM											
Plane			1	1			1			1	
	2	2				2			2		2
Dynamics	*ppp*	*ppp*	*f*	*fff*		*ppp*	*fff*		*ff*	*f*	*ff*
Density	10	20	3	6		20	6		12	6	12
Duration (second)	4	3	2	2	3	2	4	3	4	1	1
Accumulated Time (second)	263	266	268	270	273	275	279	282	286	287	288
Remark											

Table 4-2 Continued

Measure	158	$158^1/_2$	$160^1/_2$	$161^1/_2$	$162^1/_2$	$164^1/_2$	$165^1/_2$	$166^1/_2$	172	173	177
Set	$+\overline{B}\overline{C}$	$\overline{B}\overline{C}$ seul	$+\overline{AB+\overline{A}\overline{B}}$	$\overline{AB+\overline{A}\overline{B}}$ seul	$+A\overline{B}\overline{C}$	$(\overline{AB+\overline{A}\overline{B}})\overline{C}$	$(AB+\overline{A}\overline{B})C$ rappel	$(\overline{AB+\overline{A}\overline{B}})\overline{C}$	$A\overline{B}\overline{C}$ rappel	Rest	$+\overline{A}\overline{C}$
Meter									6/8		12/8
Tempo MM											
Plane	1	1			1				1		1
			2	2		2	2				
Dynamics	*f*	*f*	*ppp*	*ppp*	*fff*	*ppp*	*ff*	*ppp*	*fff*		*f*
Density	10	10	1	1	3	3 -> 5	6	5	3		10
Duration (second)	1	4	2	2	4	2	2	11	1	9	7
Accumulated Time (second)	289	293	295	297	301	303	305	316	317	326	333
Remark											

Table 4-2 Continued

Measure	181	182	$184^1/_2$	186	187	188	190	191	192	194	$195^1/_2$
Set	$\overline{(AB+\overline{\overline{AB}})}\overline{C}$	Rest	$A\overline{B}\overline{C}$ rappel	Rest	$\overline{(AB+\overline{\overline{AB}})}\overline{C}$ rappel	$(AB+\overline{AB})C$ rappel	$\overline{(AB+\overline{\overline{AB}})}\overline{C}$ seul	Rest	$\overline{AC}$	$A\overline{B}\overline{C}$ rappel	$\overline{AC}$ toujours
Meter											
Tempo MM											
Plane			1						1	1	1
	2				2	2	2				
Dynamics	*ppp*		*fff*		*ppp*	*ff*	*ppp*		*f*	*fff*	*f*
Density	5		1		1	10	1		5	3	3
Duration (second)	2	5	2	3	3	3	2	2	4	3	3
Accumulated Time (second)	335	340	342	345	348	351	353	355	359	362	365
Remark											

Table 4-2 Continued

Measure	197	199	202	203	205	206	208	210	214
Set	$+\overline{A}B\overline{C}$	Rest	$\overline{(AB+\overline{A}\overline{B})}\overline{C}$ rappel	$(AB+\overline{A}\overline{B})C$ rappel	$+A\overline{B}\overline{C}$ rappel	$+\overline{A}B\overline{C}$ rappel	$\overline{(AB+\overline{A}\overline{B})}\overline{C}$ muté sur rappel	Rest	F
Meter		6/8		12/8					
Tempo MM									
Plane	1				1	1			
			2	2			2		
Dynamics	*fff*		*ppp*	*ff*	*fff*	*fff*	*ff*		*fff*
Density	20		1	3	1	3	6		20
Duration (second)	4	3	1	4	2	4	4	8	10 (8)
Accumulated Time (second)	369	372	373	377	379	383	387	395	405 (407)
Remark									

4.1.3 Actual Deployment of Sets in *Herma*

It appears that the actual deployment of sets is different from the theoretical set contents in *Herma*. There are discrepancies between the theoretical pitches and actual deployment of pitches, including overlaps between the original set and its complement set. The numbers of elements between the theoretical and actual sets do not show specific ratios but are randomly [stochastically][74] chosen and placed. Notes that do not appear in the actual deployment are marked with crosses and additional notes that are in use but not in the theoretical operation are noted separately. No specific relations between the theoretical sets and actual deployments are found and no reasoning for actual deployment of sets was given by the composer, except the brief comment on the existence of “a stochastic correspondence”[75] between the sets and time.

Preliminary sets A, B and C are notated in Figure 4-1 to Figure 4-3, and their negations are:

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[74] Varga, 85.

[75] Xenakis, 175.

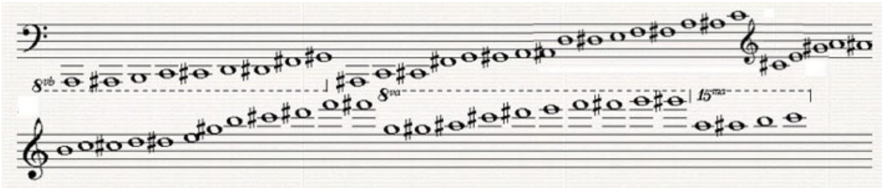

Figure 4-9: $\overline{A}$

Figure 4-10: $\overline{B}$

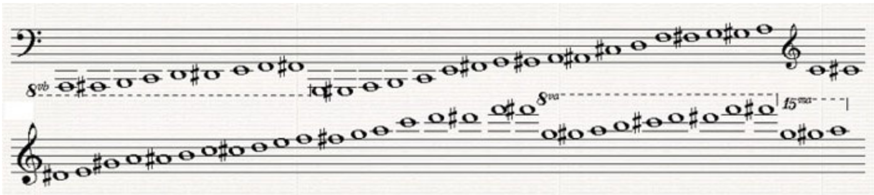

Figure 4-11: $\overline{C}$

From measure 62, $\overline{A}$ unfolds the entire pitches in the set; however, it also includes the notes that are doubled with set *A* in addition to it, as follows:

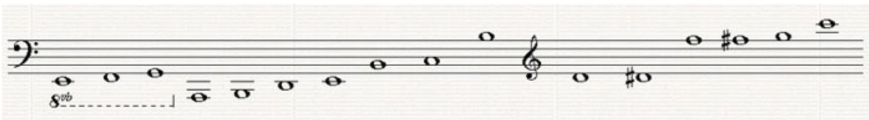

Figure 4-12: Additional Notes to $\overline{A}$

The actual deployment of $\overline{B}$ from measure 98 is missing one note (crossed out in Figure 4-13) from the theoretical set, but on the other hand it acquires 12 additional notes, which belong to set *B* (Figure 4-14).

Figure 4-13: $\overline{B}$

Figure 4-14: Additional Notes to $\overline{B}$

$\overline{C}$, starting at measure 118, also shows its discrepancy as follows in Figure 4-15:[76]

[76] Notes crossed out designate missing elements of a theoretical set when deployed throughout.

Figure 4-15: Actual deployment of $\overline{C}$

Theoretical *AB* is as given in Figure 4-16, and actual deployment of *AB* at measure 135 shows only 8 elements of the set and stays within the set (Figure 4-17):

Figure 4-16: Theoretical Set AB

Figure 4-17: Actual Deployment of AB, mm. 135-137

When *AB* is repeated once again at measure 141, F#4 is included and only one note, D3, is added to it as shown below:

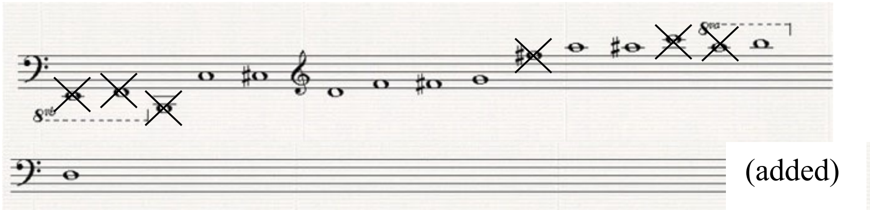

Figure 4-18: AB, mm.141-142

Operation *BC* starts by itself at measure 140 and is overlapped with *AB* at measure 141, and it repeats over $AB + \overline{AB}$ at 145$^{1}/_{2}$[77] and over $(AB + \overline{AB})C$ at 156$^{1}/_{2}$. The actual content of deployed notes differs each time:

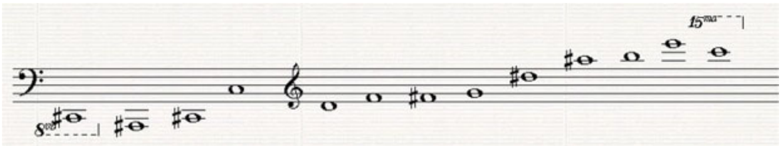

Figure 4-19: Theoretical Set BC

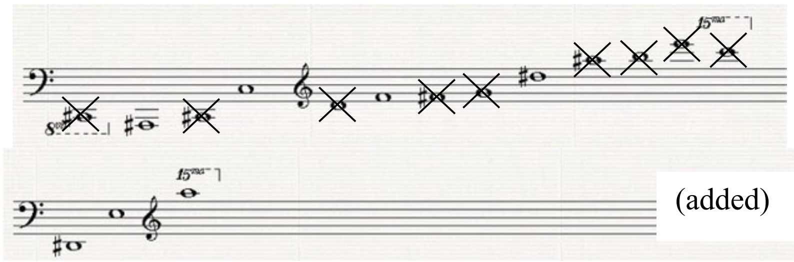

Figure 4-20: Actual Deployment BC, m. 140

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[77] Measure numbers including $^{1}/_{2}$ mean the set begins at the second half of the given measure.

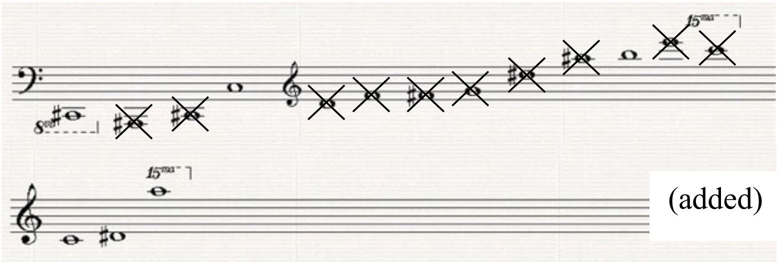

Figure 4-21: BC, m. $145^{1}/_{2}$

Figure 4-22: BC, m. $156^{1}/_{2}$

The content of operation *AC* is given below: however, because *AC* is not a part of the given operations used (Figure 4-7), it is never deployed in the actual piece.

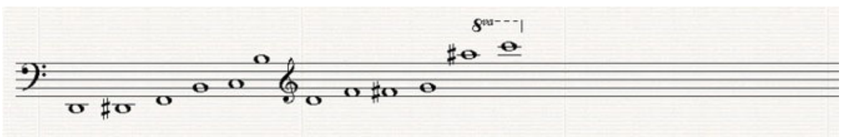

Figure 4-23: Theoretical Set AC

$\overline{AB}$ does not appear by itself, but always as a union with *AB*. The theoretical content of $\overline{AB}$ is:

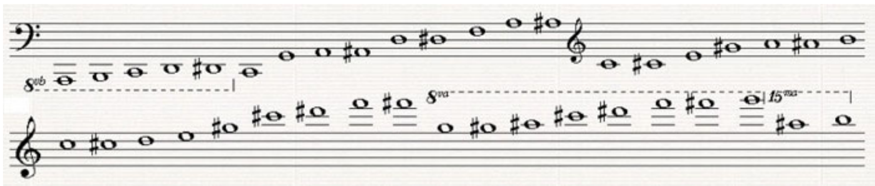

Figure 4-24: Theoretical Set $\overline{AB}$

Therefore $AB+\overline{AB}$ is consist of the following:

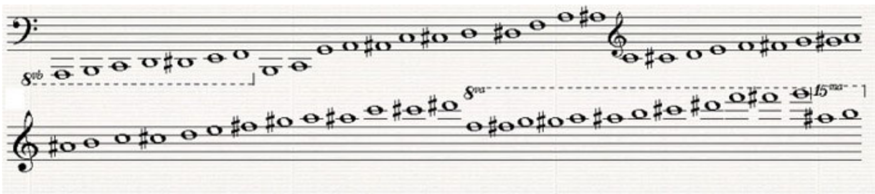

Figure 4-25:AB+ $\overline{AB}$

$AB+\overline{AB}$ is used twice, once at measure 143 and the other, repeat, at measure 150. The actual contents are different as shown in Figure 4-26 and in Figure 4-27.

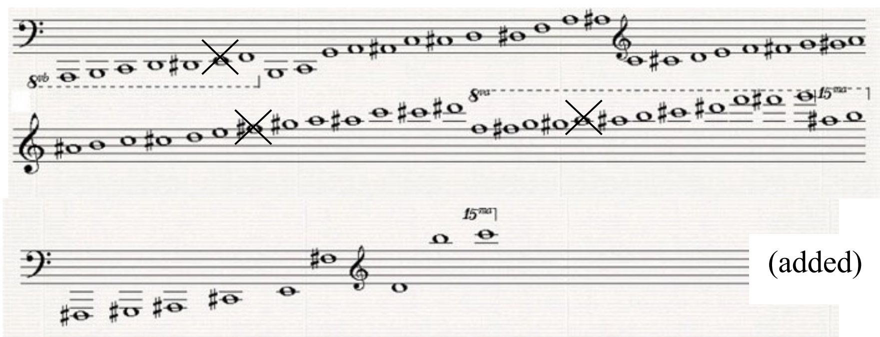

Figure 4-26: AB+ $\overline{AB}$***, m. 143***

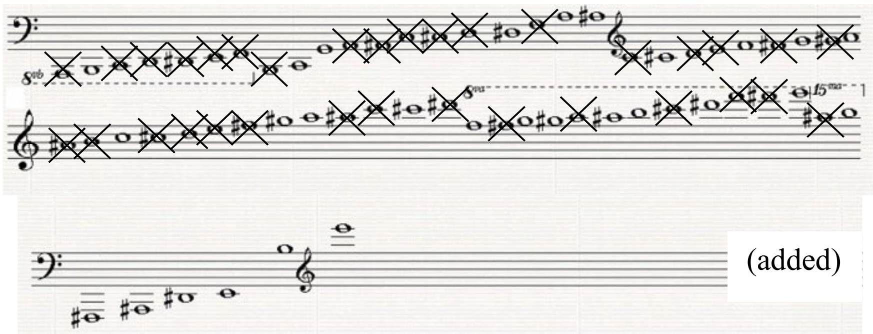

Figure 4-27: AB+$\overline{AB}$, m. 150

The complement set of AB+$\overline{AB}$ at measure 160 is comprised of 33 notes based on the operation, but 11 notes from the set are used with an additional 5 notes outside of the theoretical set.

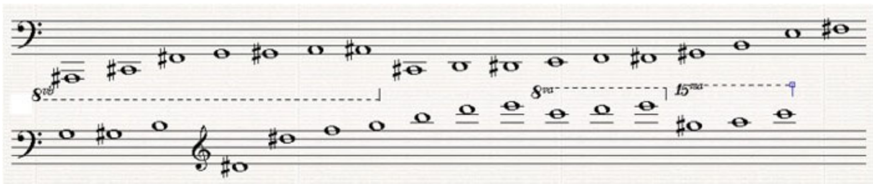

Figure 4-28: Theoretical Complement Set of AB+$\overline{AB}$

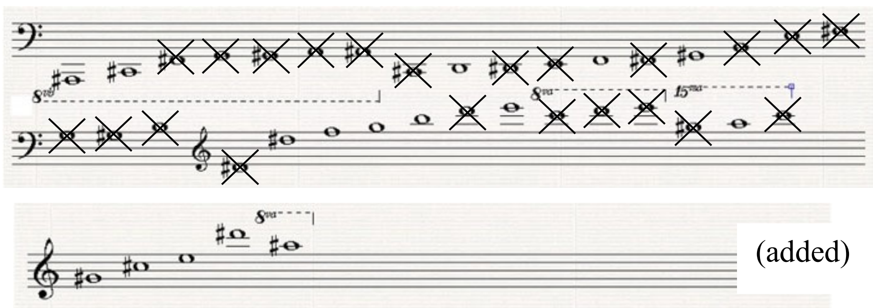

Figure 4-29: Complement Set of AB+$\overline{AB}$

The theoretical set $(AB+\overline{AB})$C is comprised of 13 notes as shown in Figure 4-30 and appears four times throughout the piece at measure 155 with 12 notes from the theoretical set and 12 notes outside set, at measure $165^{1}/_{2}$ with six notes from the theoretical set and two outside, at measure 188 with nine theoretical notes and three additional, and at measure 203 with six and two notes just as at measure $165^{1}/_{2}$.

Figure 4-30: $(AB+\overline{AB})$C, Theoretical Set

Figure 4-31: $(AB+\overline{AB})$C, m. 155

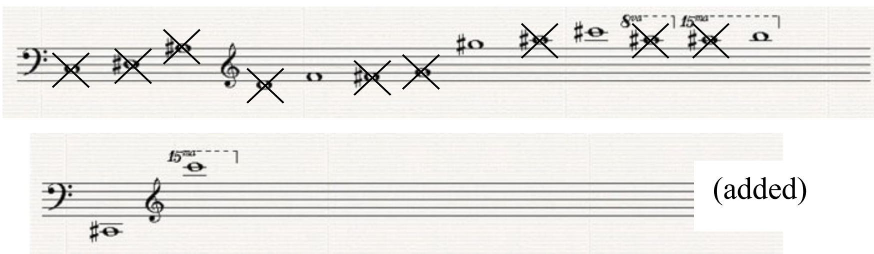

Figure 4-32: $(AB+\overline{AB})$C, m.$165^{1}/_{2}$

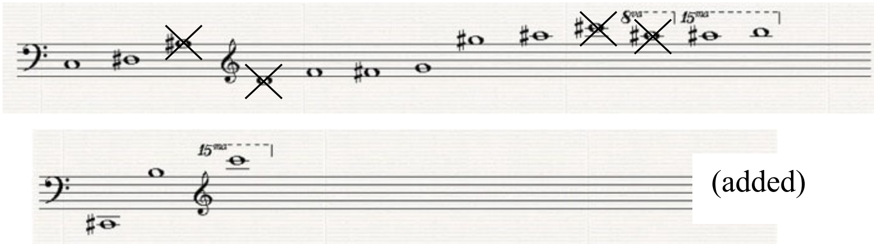

Figure 4-33: (AB+ $\overline{A}\overline{B}$)C, m. 188

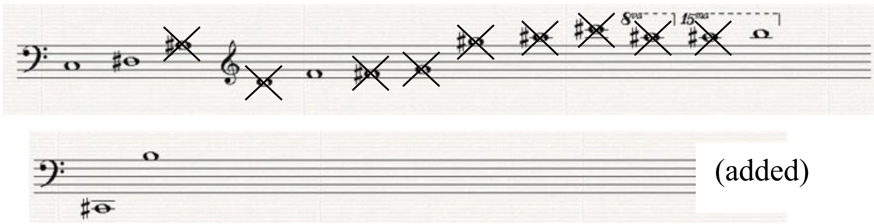

Figure 4-34:* $\overline{(AB+\overline{AB})}\,\overline{C}$ *, m. 203

$\overline{(AB+\overline{AB})}\,\overline{C}$ appears five times at measures at 164, 181, 187, 202 and 208. Notes are used in common for up to four appearances; however, not one note was common for all of the five places. The theoretical set and the deployments are as shown below:

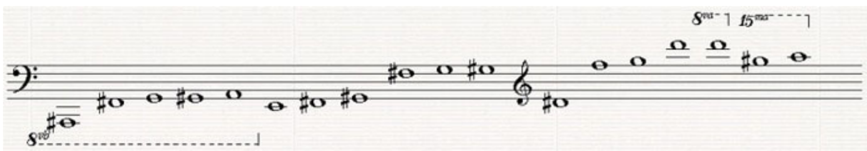

***Figure 4-35: Theoretical Set of* $\overline{(AB+\overline{AB})}\,\overline{C}$**

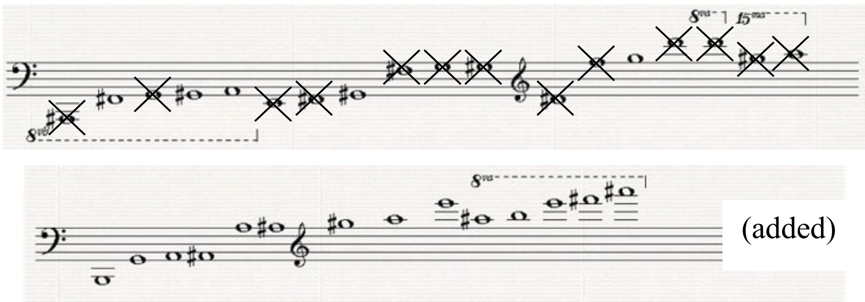

Figure 4-36: $(\overline{AB + \overline{\overline{AB}}})\overline{C}$ ***, m. 164***

Figure 4-37: $(\overline{AB + \overline{\overline{AB}}})\overline{C}$ ***, m. 181***

Figure 4-38: $(\overline{AB + \overline{\overline{AB}}})\overline{C}$ ***, m. 187***

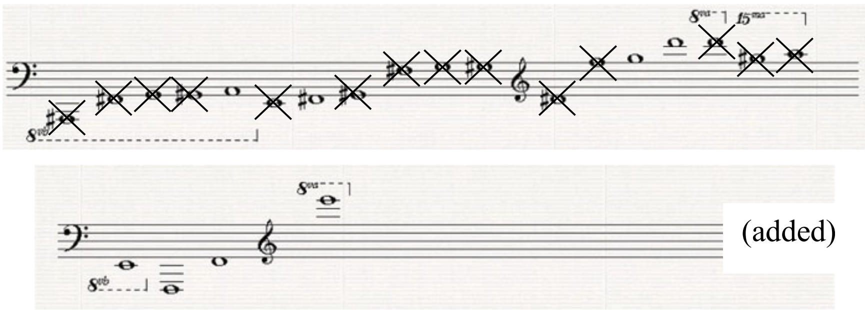

Figure 4-39: $\overline{(AB+\overline{AB})}\,\overline{C}$***, m. 202***

Figure 4-40: $\overline{(AB+\overline{AB})}\,\overline{C}$***, m. 208***

Operation *ABC* is used only once at measure 146. Only four notes of the theoretical set are used and the other three notes deployed are from outside the set.

Figure 4-41: Theoretical Set ABC

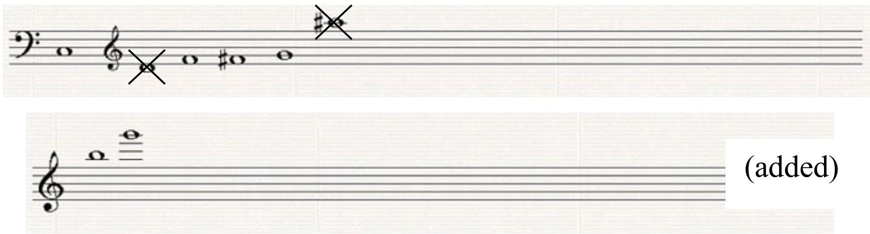

Figure 4-42: ABC, m. 146¹/₂

Theoretical set $\overline{A}\overline{B}C$ appears once at measure 151, and the theoretical set and actual deployment are as follow:

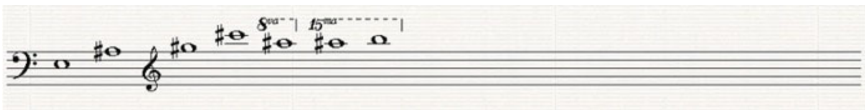

Figure 4-43:Theoretical Set of $\overline{A}\overline{B}C$, m. 151

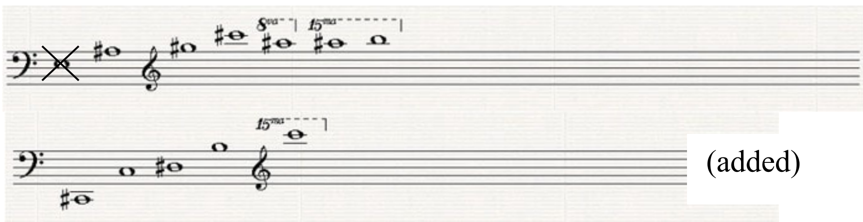

Figure 4-44: $\overline{A}\overline{B}C$, m. 151

The theoretical sets and actual deployments of set $A\overline{B}\overline{C}$ at measures $162^1/_2$, 172, $184^1/_2$, 194 and 205, and set $\overline{A}B\overline{C}$ at measures 197 and 206 are given below. Additional notes of set $A\overline{B}\overline{C}$ at measures $184^1/_2$ and 194 are same as shown in Figure 4-48 and Figure 4-49.

Figure 4-45: Theoretical Set of $A\overline{B}\overline{C}$

Figure 4-46: Actual Deployment of Set $A\overline{B}\overline{C}$, m. $162^1/_2$

Figure 4-47: $A\overline{B}\overline{C}$, m. 172

Figure 4-48: $A\overline{B}\overline{C}$, m. $184^1/_2$

Figure 4-49: $A\overline{B}\overline{C}$***, m. 194***

Figure 4-50: $A\overline{B}\overline{C}$***, m. 205***

Set $\overline{A}B\overline{C}$ deploys twice at measure 197 and 206 with three common notes from the theoretical set and two common additional notes; at measure 197, however, two more are added from outside the theoretical set.

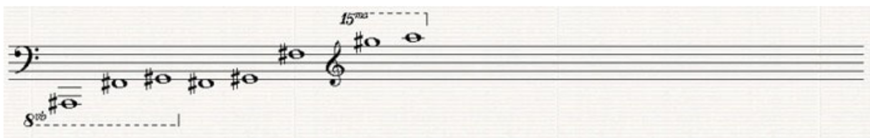

Figure 4-51: Theoretical Set of $\overline{A}B\overline{C}$

Figure 4-52: $\overline{A}B\overline{C}$ ***, m. 197***

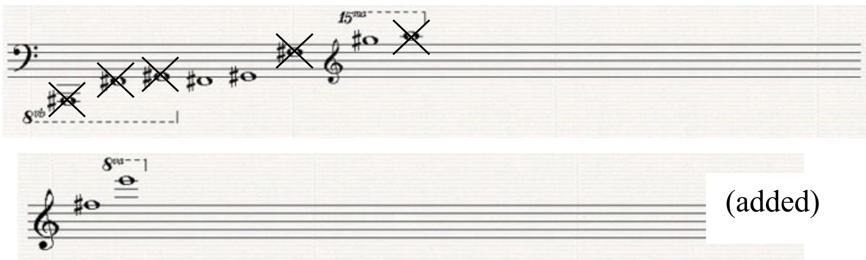

Figure 4-53: $\overline{A}B\overline{C}$ ***, m. 206***

The final operation, function F[78] starting at measure 214, is comprised of 30 notes theoretically, however, only 21 notes from the theoretical set and 16 additional notes are actually deployed.

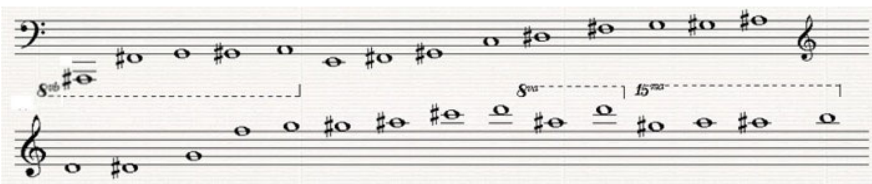

Figure 4-54: Theoretical Set F

[78] $F = ABC + A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C = (AB + \overline{A}\overline{B})C + (\overline{AB + \overline{A}\overline{B}})\overline{C}$

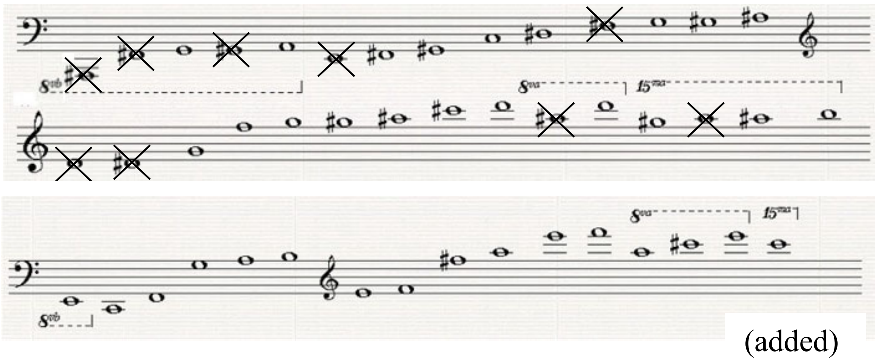

Figure 4-55: Actual Deployment of F

4.1.4 Proportions in *Herma*

Mean densities given for set R, in Figure 4-8, show that different density values were applied to deploy notes, and the densities read:

Density ($^{s}/_{sec}$)	1.73	2.80	4.53	7.32	11.8	19	31

The starting value 1.73 is obtained by quarter notes at the given tempo MM=104: $104 \div 60sec = 1.73\ ^{s}/_{sec}$.

The sum of two consecutive densities becomes the following density; the sum of 1.73 and 2.80 becomes 4.53, the sum of 2.80 and 4.53 equals 7.32, and likewise (Figure 4-56, numbers are approximate).

$$
\begin{array}{ccccccccccccc}
1.73 & + & 2.80 & = & 4.53 & & & & & & & & \\
 & & 2.80 & + & 4.53 & = & 7.32 & & & & & & \\
 & & & & 4.53 & + & 7.32 & = & 11.8 & & & & \\
 & & & & & & 7.32 & + & 11.8 & = & 19 & & \\
 & & & & & & & & 11.8 & + & 19 & = & 31
\end{array}
$$

Figure 4-56: Structure of Densities in set R

Densities given above are close to the first seven numbers of the Fibonacci series as well (Figure 4-57).

Density ($^{s}/_{sec}$)	1.73	2.80	4.53	7.32	11.8	19	31
Fibonacci series	2	3	5	8	13	21	34

Figure 4-57: Fibonacci Numbers and Densities in set R

The number of events for whole R is 208 and set R is comprised of seven sections with different mean densities; therefore, the mean density for each section is 29.7: $208\ s \div 7 = 29.7\ s$.[79]

Approximately the 30th note appears when the tempo changes to MM=120, wherein one second equals a half note. Based on the theoretical calculation, the portion with δ=1.73 lasts for approximately 17 seconds, which means 29.7 sound events: $17\ sec \times 1.73\ ^{s}/_{sec} = 29.7\ s$, the next portion with δ=2.80 for 10 seconds, which also results in 29.7 sound events and the rest of set R follows this principle accordingly; therefore, the following calculation is possible (Figure 4-58):

Density ($^{s}/_{sec}$)		1.73	2.80	4.53	7.32	11.8	19	31
Duration (sec)	X	17	10	6.5	4	2.5	1.5	0.9
	29.7							

Figure 4-58: Mean Density

The ratio between two consecutive durations follows the golden proportion (Figure 4-59).

Duration (sec)	17	10	6.5	4	2.5	1.5	0.9
Ratios	0.59	0.65	0.65	0.625	0.6	0.6	

Figure 4-59: Durational Ratios

[79] See ibid. 22-37, for more detailed reasoning and examples.

However, actual deployment shows a little difference in terms of duration as follows.[80] The durations follow the Fibonacci series, which in fact shares the same principle with the golden section, in reverse order (Figure 4-60):

Density (s/sec)	1.73	2.80	4.53	7.32	11.8	19	31
Duration (sec)	13	8	5	3	2	1.2	0.7
Fibonacci Series	13	8	5	3	2	1	1

Figure 4-60: Fibonacci Numbers and Durations in set R

The golden section point (GSP hereafter) of the whole piece should appear at second 250 (out of 405 seconds) and the deployment of individual sets *A*, *B* and *C* is completed at second 247 in actual calculation of the time. At this point, the operation of the sets begins based on the given formula (Figure 4-61).

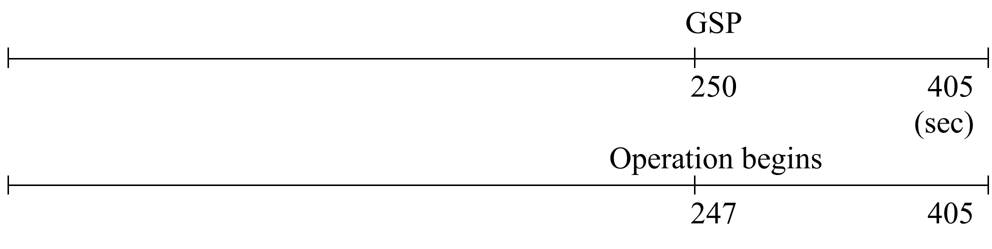

Figure 4-61: GSP of the whole piece

[80] Calculated based on the given tempo MM=104 for the first six measures, and MM=120 for the rest.

When the first 33 seconds of set R are excluded from calculation, the GSP appears at second 263, where the operations of more than two sets begin with $AB + \overline{A}\overline{B}$ with density of 10 (Figure 4-62).

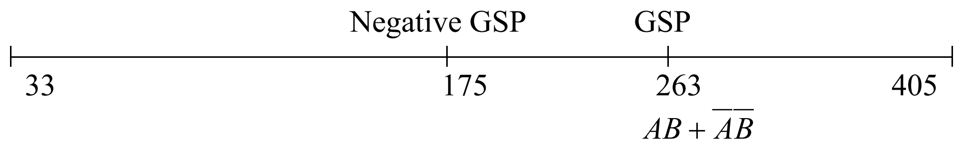

Figure 4-62: Sec. 33 – 405

Negative GSP of the portion from second 33 to the end is at second 175, which is also the positive GSP of the portion of seconds 33 to 263, and the golden mean from 33 to 175 appears at second 119. At this point, the deployment of sets that are related to A (including $\bar{A}$) ends and set B starts. This, second 119, also becomes the negative golden section of the portion of seconds 33 to 263 (Figure 4-63).

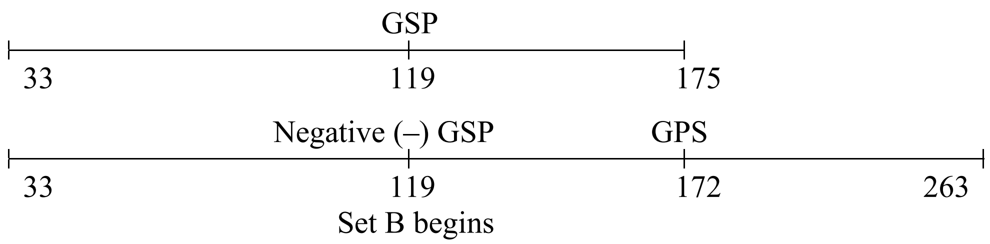

Figure 4-63: Sec. 33 – 175, sec. 33 – 265

The GSP of seconds 33 to 395 appears at second 257 and the negative at 171, when the last 10 seconds of function F is also excluded from calculation. The negative GSP 171 is close to 172, the GSP of the portion from 33, where the individual sets begin, to 263, where both union and intersection start to be in use (Figure 4-64). At seconds 257 and 171 entries of different sets or operations appear after major rests: BC after four seconds of rest at 257, $\overline{B}$ at 171 after eight seconds of rest (Figure 4-64).

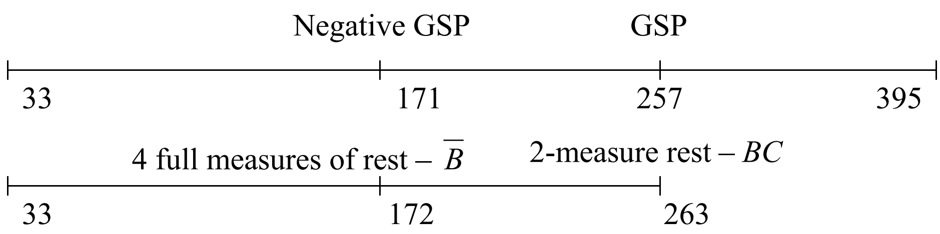

Figure 4-64: Sec. 33 – 395, sec. 33 – 263

Although not every entry of sets following measures of rest shows the evidence of the golden section in time, it seems that the majority of such cases follows the golden section. The entry of $\overline{A}$ after 4 seconds (2 measures) of rest meets with the negative GSP of the portion from the beginning to the GSP of the whole piece, at second 96. The GSP 154 of this portion approximates 155 of the negative GSP of the whole piece.

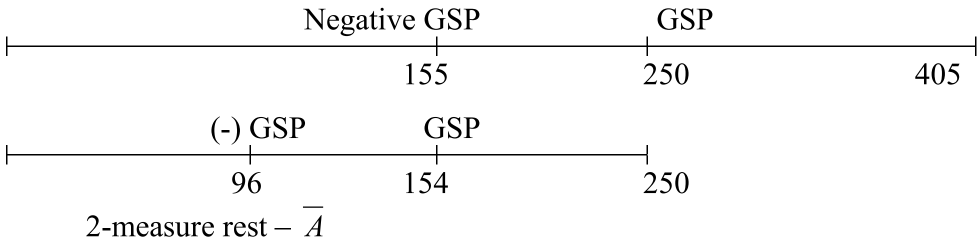

Figure 4-65: – sec. 250, – sec. 405

Another example of the entry after a rest appears at second 340, where the repeat of *ABC* occurs. This point is the GSP of the shorter [negative] golden section of the whole piece (after second 250). Also at second 345, where $\overline{(AB+\overline{AB})}\overline{C}$ repeats, is the GSP of the portion after 247, where the operation of sets begins (Figure 4-66).

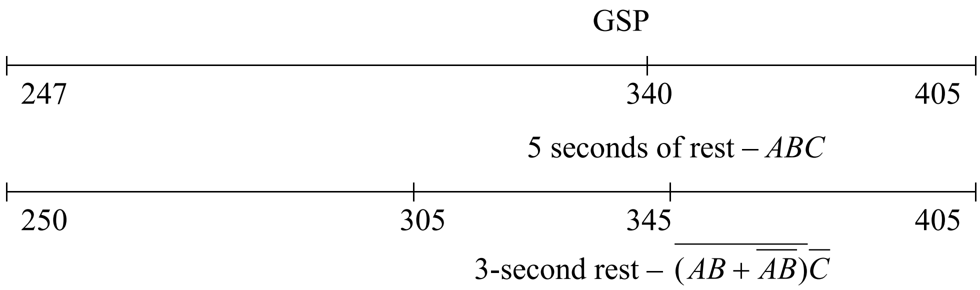

Figure 4-66: Sec. 250 - 405. Negative Golden Portion of the Whole Piece

The midpoint of the section above, from seconds 247 to 405, appears at second 326, which is the entry of the operation $\overline{C}$. This follows the longest rest of the piece (9-second rest). The negative GSP shown in Figure 4-66, second

305, leaves 100 seconds to the end, and therefore the midpoint thereafter is at 355, where another entry of $\overline{AC}$ following a rest measure occurs (Figure 4-67). This shows that not only golden proportions but also symmetrical balance can be identified in *Herma*.

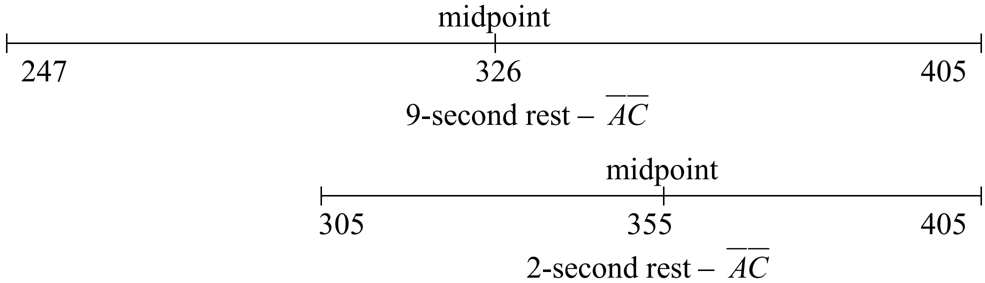

Figure 4-67: Midpoints of sec. 247 – 405, sec. 305 – 405

326 also corresponds with the mid-point of 257 and 295, which is the shorter golden portion of 33 to 395 (Figure 4-68).

Figure 4-68: GSP and Midpoints of sec. 33 – 395, sec. 247 – 405

Set *A* deploys from seconds 33 to 119, set *B* 119 to 193, and set *C* 193 to 247. GPS's of the three sets appear at 86, 164, and 226 seconds, and negative points at 66, 154, and 214, respectively. The durational proportion of set *A* and *C* are seen from Fibonacci Series, 55 and 89[81]; therefore, golden portions of set *A* and set *C* show approximate Fibonacci numbers, 8, 13, 21, 34 and 55 (Figure 4-69).

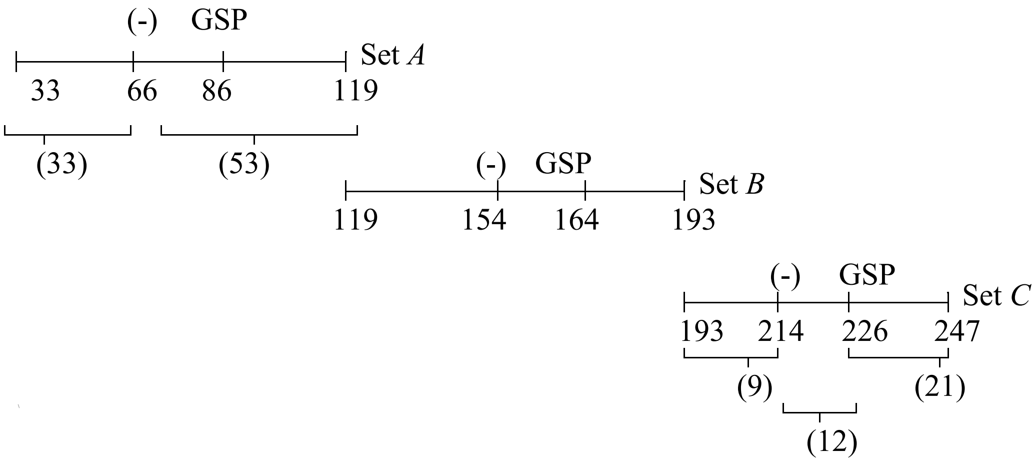

Figure 4-69: Durational Ratios of Sets A, B and C

The negative GSP of set C, second 214, is also the mid-point of the main body of the piece, which excludes set R and function F, from 33 to 395. The same duration of set C is obtained from 138 to 193, which makes 193 a mid-point; at 137 appears an entry of a set after a rest, and this section is

[81] Actual calculation results in 86 seconds instead of 89. However, the numbers are approximate for most of the cases, for golden proportion itself is an interminating decimal (0.6180339887…).

divided into two parts with the same durations at second 165, which is close to the GSP of set *B*'s deployment.

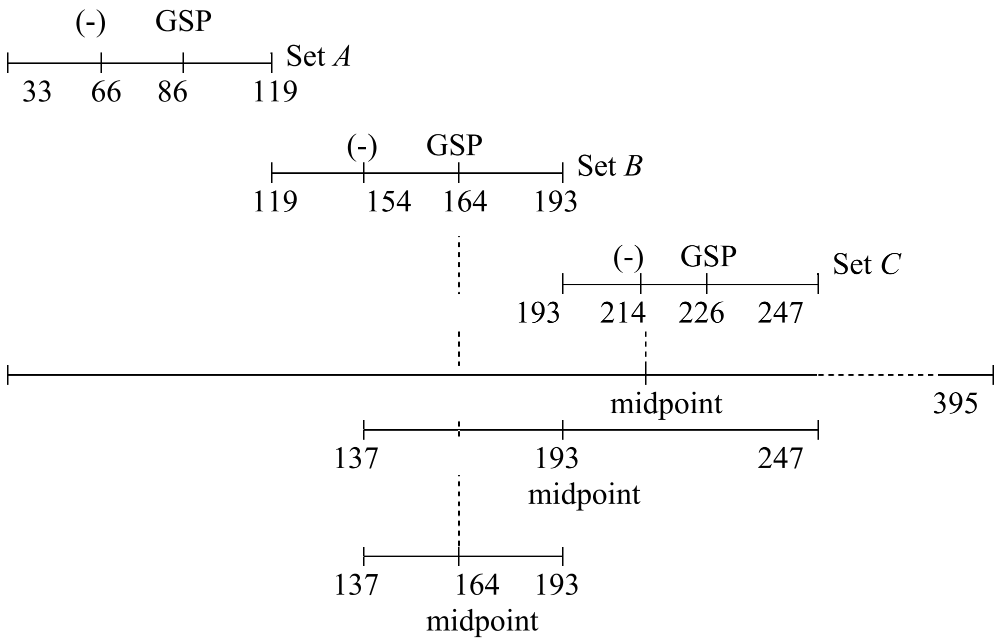

Figure 4-70: GSP's and Midpoints of Sets A, B and C

The densities given for the sets *A*, *B*, and *C* cooperate with duration to create golden proportion and symmetrical balance as well. Set A is unfolded in three different ways: *A* linéaire, *A* nuage, and *A* compliment (Ā). The number of notes theoretically deployed is 454.2, calculated based on the density and duration given by composer.

A linéaire $0.8\ (^{s}/_{sec}) \times 60\ (sec) + A$ nuage $3.3 \times 4 + 5 \times 28 + \overline{A}\ 10 \times 22 = 454.2$, and actual notes used are 445. A linéaire appears to actually have 48 notes in composition, A nuage 28 and 150 each, and $\overline{A}$ 205, instead of theoretical numbers of 48, 46, 140, and 220 respectively. This seems to show that although the pre-compositional plans are set up in a rather strict and complex way, adjustments were made during the actual compositional process for musical and aesthetic reasons.

Set B also shows the difference in numbers of notes between the calculation and actual notation. First B linéaire, of which actual notes are 27, theoretically consists of 28.8 notes ($1.8\ (^{s}/_{sec}) \times 16$ (sec) = 28.8 (s)), and B nuage with actual number of notes 44 theoretically result in 46.2 notes ($3.3(^{s}/_{sec}) \times 14$ (sec) = 46.2 (s)). The number of notes of B nuage that juxtapose B linéaire makes the positive golden proportion from the number of B linéaire notes: 27 (notes) $\div$ 44 (notes) = 0.614; 28.8 (notes) $\div$ 46.2 (notes) = 0.62. Therefore, the total theoretical number of notes of B linéaire is: $1.8\ (^{s}/_{sec}) \times 16\ (sec) + 5 \times 2 + 5 \times 14 = 108.8$; of B nuage is: $3.3\ (^{s}/_{sec}) \times 4\ (sec) + 5 \times 20 + 10 \times 18 = 326.2$.

The ratio of 108.8 to 326.2, which is 0.33, is close to the negative golden ratio (0.381). However, the actual number of notes of set B, 381, shows an interesting fact. The durational proportion between set A and B, which is 86 seconds : 74 seconds = 1.16 : 1, equals the ratio of the numbers of notes of the two sets, 445 notes : 381 notes = 1.16 : 1. This shows that, although the duration in the piece is not directly related to the number of actual notes, by applying the same proportional value for both time and number of notes, a sense of balance and relativity is provided for the listener.

The deployment of set *C* nuage starts with density 2.5, which makes the calculated number of notes 45. The following *C* linéaire, theoretically, has 50 notes for 10 seconds with the density of five. The complement of set *C* ($\overline{C}$) is comprised of 252 notes according to calculation (9 ($^{s}/_{sec}$)$\times$28 (sec) = 252 (s, notes)), and therefore the sum of notes of set *C* becomes the negative golden proportion of $\overline{C}$: 95 (notes)$\div$252 (notes) = 0.38.

The texture of each set becomes denser and the rhythmic movement gets faster towards the ends of sets, especially at the end of set *C* (second 233) which employs five to six voices. This seems to show the relativity of the deployment of set *A*, *B* and *C* with the scheme of the whole piece; set *A* starts in a quite sparse texture, and so does operation *AB* at second 135. Set *C* ends very thick and so do set *R*, which moves from *ppp* to *fff* and from half notes to sixteenth-note triplets with grace notes, and the function *F*, which results from all the operations at the end of the piece.

4.2 *EVRYALI*

Xenakis's second piano solo piece *Evryali*, "wide sea/Medusa,"[82] was composed in 1973. This is the first piece in which Xenakis used the arborescence method, of which the details follow below. The rhythmic value is mostly sixteenth and, when other values are used together, they keep 1:2 ratios throughout, for example quarter notes to eighth notes or eighth notes to sixteenth notes. It consists of up to eight individual voices that often reach the highest and lowest register of the piano keyboard. The voices are spread in four staves at times, based on their polyphonic voice leading rather than their chordal motions.

4.2.1 The Method of Arborescence

A group of points can form a curvy line and the continuous movement of the line forms a shape by reproducing itself at any point; a shape of a bush or even a tree: arborescence. Xenakis brought in the concept of arborescence to composition for the purpose of creating continuity from the moment of conception. Curvy lines could be drawn on a page and these lines would translate into lines of pitches. Arborescence was the basis of *Evryali* (Figure

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[82] Matossian, 233.

4-71). The first sketch of *Evryali* was drawn on grid paper with curvy lines, the arborescences.

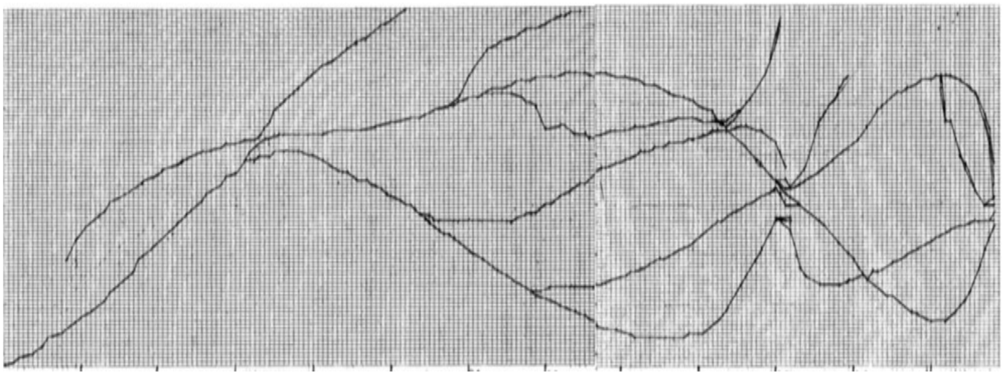

Figure 4-71: Arborescence

Evryali appears to have four significantly different materials: arborescence, wave, block, scattered sound and silence. Arborescence designates sections where the polyphonic lines follow the arborescence method (Figure 4-72). Some sections show longer, denser and more complicated arborescences, and others show shorter and simpler ones. When voices make wavy forms by moving in the same directions, it is categorized as wave (Figure 4-73). In other words, waves occur when all voices move in similar motion.

Figure 4-72: mm. 75 – 79; Arborescence

Figure 4-73: mm. 45 – 50; Wave

Some sections are comprised only of static chords, stopping the flowing motion of arborescences or waves abruptly. Blocks include both chords that are comprised of voices with irregular repeats and omission of notes (Figure 4-74) and chords that are fixed in terms of notes but change rhythm (Figure 4-75).

Figure 4-74: mm. 90 – 91; Block

Figure 4-75: mm. 88 – 89; Block

There are some measures where the phrases are fairly short (Figure 4-76) or where the individual voices and overall texture do not form any specific one of the above (Figure 4-77); these are categorized as scattered sound. Silence designates obvious long pauses, named "silence" by the composer with their durations in seconds (Figure 4-78).

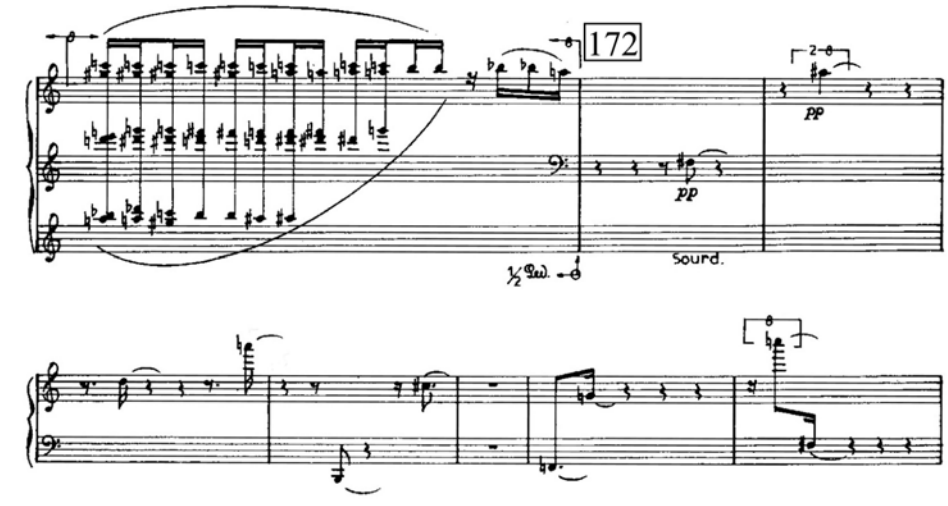

Figure 4-76: mm. 172 – 178; Scattered Sound

Figure 4-77: mm. 137 – 138; Scattered Sound

Figure 4-78: m. 65; Silence

Arborescence, in either a big or a small shape, constructs the majority of the piece. The major arborescences are used at measures 75 – 135, alternating with the blocked shapes, at 179 – 188, at 207 – 219, and fragments of arborescence and mixed arborescences with blocked chords at other places.

One of the major arborescences, measures 75 – 87, can be drawn on a grid as follows (Figure 4-79). The horizontal axis on the graph is time, and the vertical axis is pitch. One column in time designates a sixteenth-note value and one row in pitch a half note. The graph starts at measure 75 on time axis, at the lowest A on pitch axis:

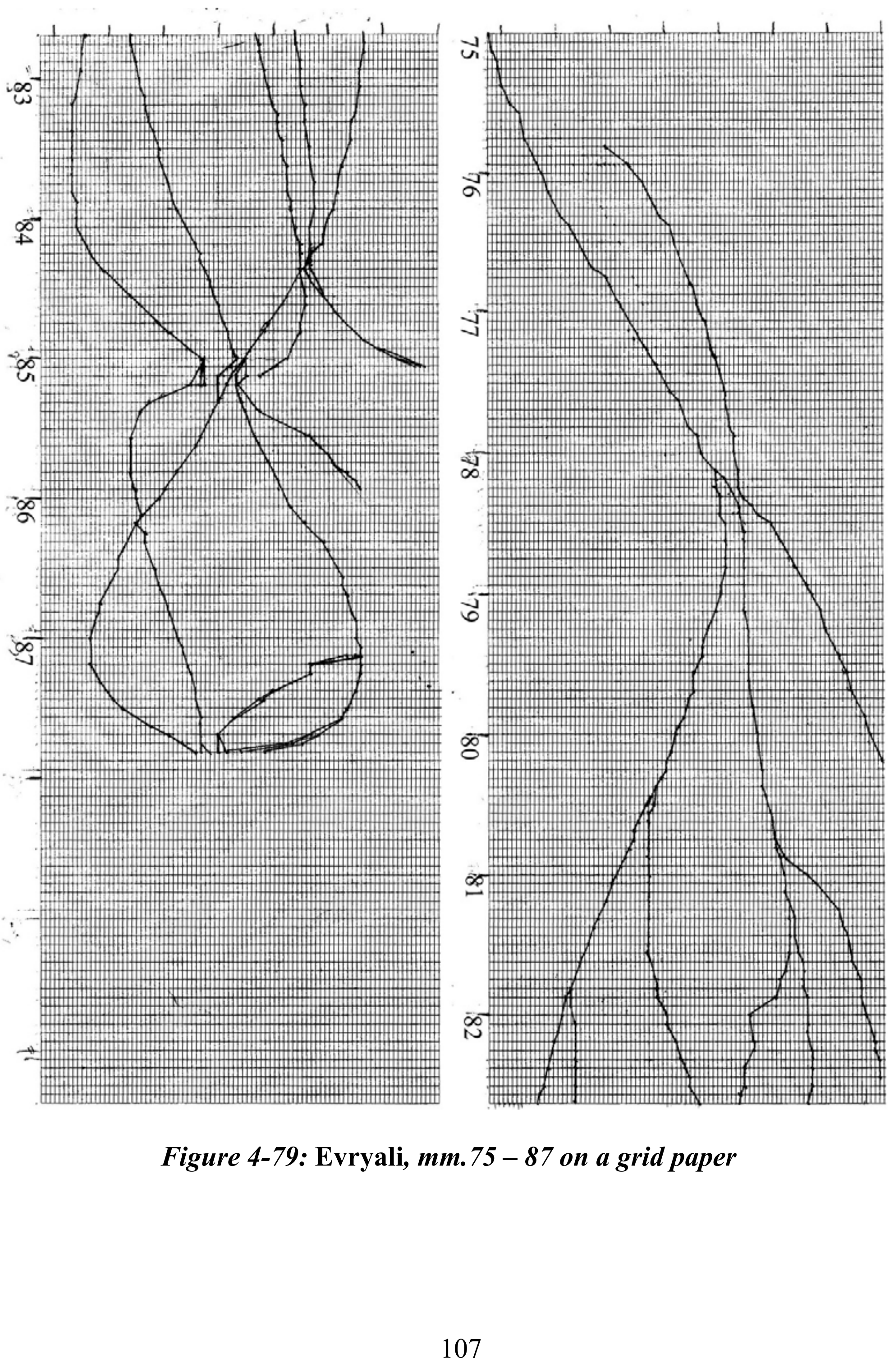

Figure 4-79: **Evryali*****, mm.75 – 87 on a grid paper***

4.2.2 Proportions in *Evryali*

Evryali is comprised of three balanced sections, with a short coda. The first section A, up to measure 74, mainly unfolds the waves, section B, from measure 75 to 135, develops arborescences, and the rest is section C, followed by a distinctive coda at measure 220 after a long silence.

A	B	C	Coda
1 – 74	75 – 135	136 – 219	220 - end

When the length of the piece is considered (excluding the coda), the positive golden section point (GSP) appears at measure 136 and the negative at measure 83.

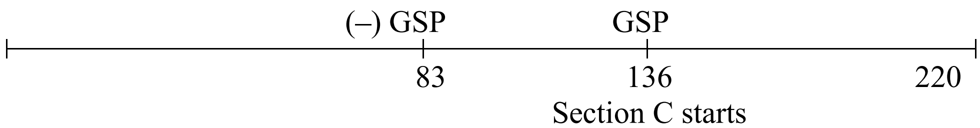

Figure 4-80: GSP of the piece

Within section A, the positive and negative GSPs are at measures 46 and 28.5, respectively. The first big wave starts at measure 46 and lasts for 15 measures. At measure 28.5 (middle of measure 28) the third phrase of the small arborescences begins.

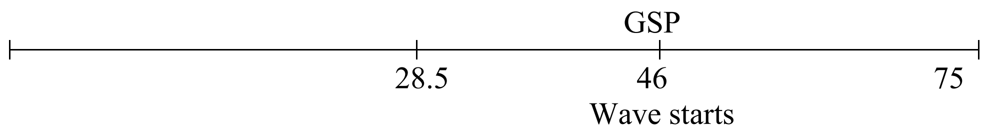

Figure 4-81: GSP of Section A

The GSP of 28.5 is 17 and at measure 17, where the GSP of the first 28.5 measures appears, the first of the small arborescences begins. Measures 29 and 17.5 (close to 28.5 and 17) are the positive and negative GSPs, respectively, when 46 measures (up to the GSP of section A) are considered.

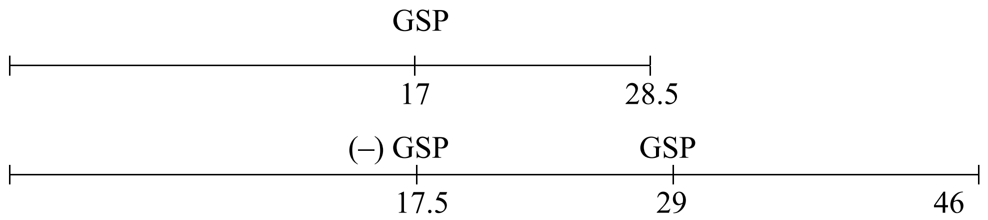

Figure 4-82: Section A, mm. 1 – 46

Measure 46, which is the GSP of section A, becomes the negative GSP when the 46.5 measures, from measures 28.5 to 75, are considered. The first big wave motion begins at measure 46 as mentioned before. The positive GSP of this portion is at measure 57, where the wave motion, which starts at measure 46, hits the lowest register for the second time.

Figure 4-83: Section A, mm. 28.5 – 75

Measure 57 also becomes the negative GSP of the negative golden portion of section A (measures 46 – 75). The GSP of this case is at measure 64 and here ends the first use of block passages. The next block begins at measure 70, which is the GSP of the part from measure 64, the GSP above, to measure 75.

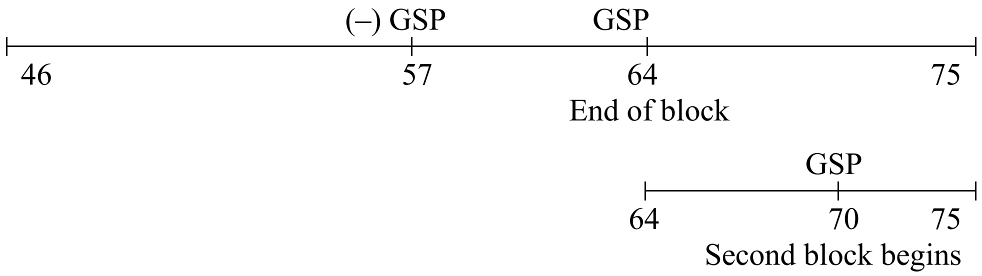

Figure 4-84: Section A, mm. 46 – 75

The overall proportional relations are as below:

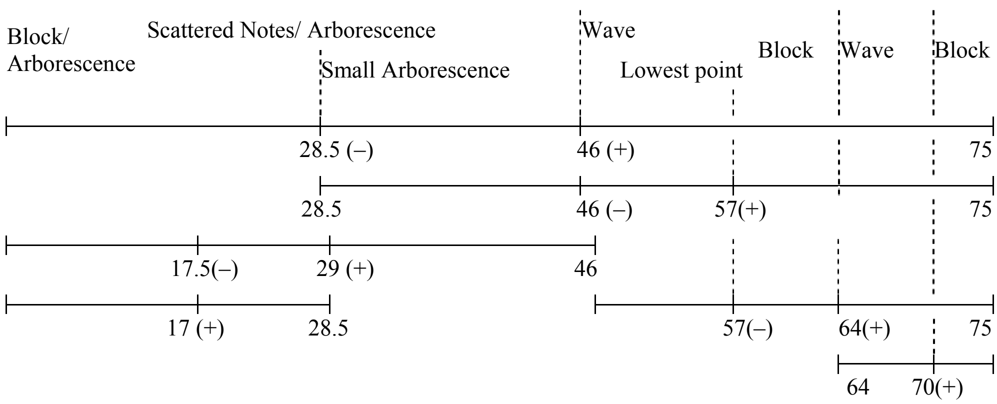

Figure 4-85: Section A, Proportions

Section B, from measure 75 to measure 136, is comprised of 62 measures and therefore the GSP of section B is at measure 113 and the negative GSP at measure 98.

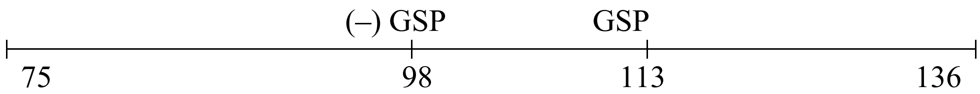

Figure 4-86: Section B

The majority of section B is arborescence, however, five major blocked chords are also in use in section B. The blocks and arborescences are either juxtaposed or superimposed. The starting points of one of the arborescences and one of the blocks are at both negative and positive GSPs of section B, respectively, although the actual occurrence of the arborescence around the negative GSP is off by one measure, at measure 97.

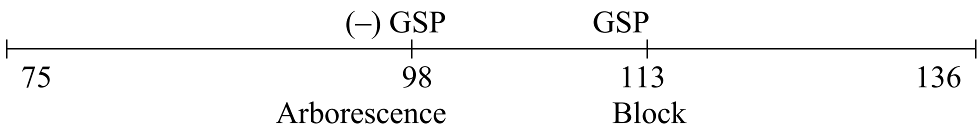

Figure 4-87: Section B

The three parts, which are divided by two GSPs, can also be subdivided within by the golden mean. The positive GSPs of the three subdivisions of section B are at measure 89, measure 107 and measure 127, and the first two correspond with two of the starting points of the blocked patterns: at measure 88 and measure 107. The negative GSPs are at measure 83, measure 103 and measure 121, correspondingly. At measure 102, which is close to 103, a long arborescence of section B occurs, which is superimposed twice by blocks towards the end of the section. This long arborescence starts to split into two opposite directions at measure 121.

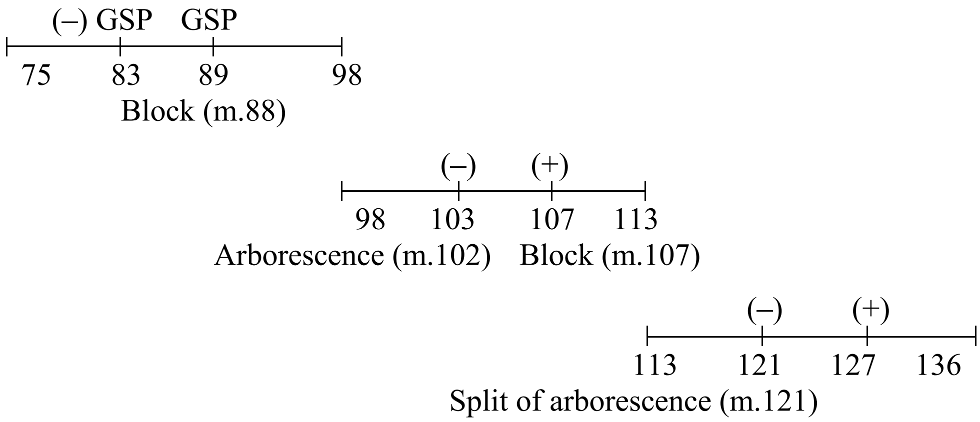

Figure 4-88: Section B

Also note that the first negative GSP of Figure 4-88 is the negative GSP of the whole piece. When the portion of measures 83 – 136 is considered, the negative GSP is at 103, which is also seen in Figure 4-88, as one of the negative GSPs.

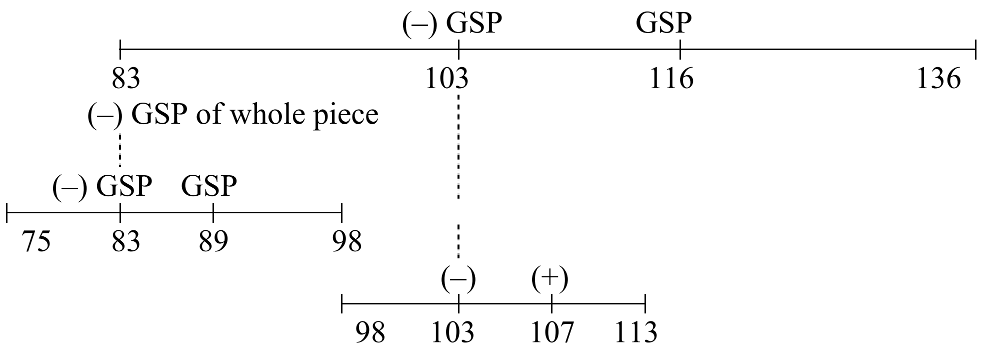

Figure 4-89: Section B

The negative and positive GSPs are at 113 and 121 respectively, when the longer golden portion of section B from the negative GSP 98 is considered. 89 and 98 are the negative and positive GSPs of the part from measure 75 to measure 113, which is the GSP of section B. It appears that section B also shows an inter-relationship among GSPs of the subdivided parts.

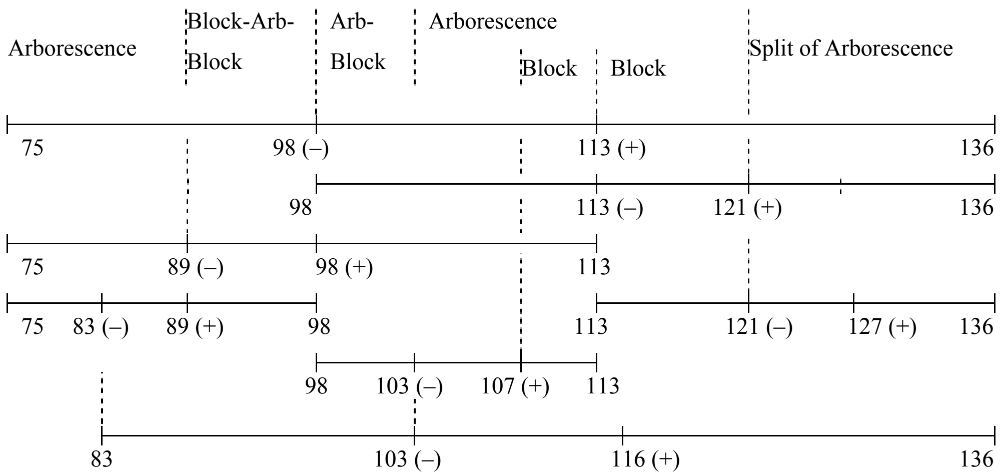

Figure 4-90: Section B, Proportions

Section C also appears to have both macro- and microstructure that are proportioned by golden section. Section C starts with scattered notes with short and thin arborescences, which become longer towards the middle, and a long, thick group of blocks is followed by arborescence towards the end. The GSP of section C is at measure 188 and the negative GSP is at measure 168.

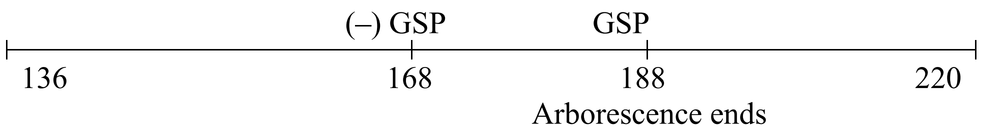

Figure 4-91: Section C, GSPs

As shown in Figure 4-91, the positive GSP 188 meets with the point where the long descending arborescence ends before a six-second pause.

The positive and negative GSPs of measures 136 – 168 are at 156 and 148, respectively. Very thin and short arborescences begin at measure 147 and this approximates the negative GSP of measure 136 – 168. These short arborescences, which start at measure 148, cease at measure 172 and resume at measure 179.5, and measure 179.5 is close to the GSP of measures 168 – 188, which is 180. At measure 176, the negative GSP of measures 168 – 188, appears a rest measure.

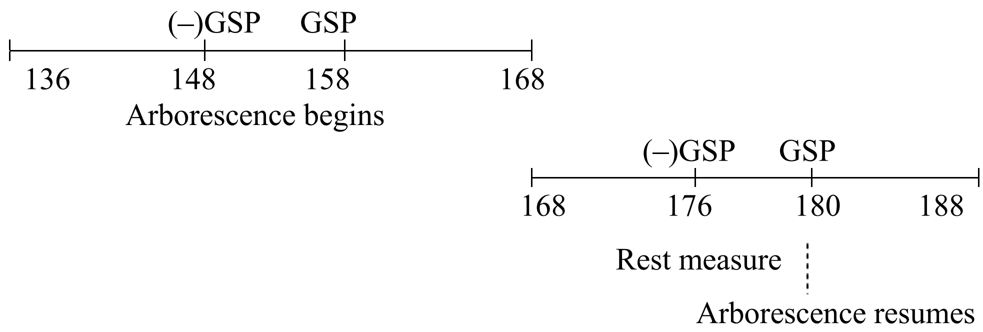

Figure 4-92: Section C, m.136 – 168, m.168 – 188

The rest of section C, from measure 188 to 220, consists of blocks and arborescence. The longest blocked pattern begins at measure 190: the blocks start from both ends of the keyboard of the piano and they move inwards to finally merge at measure 199, around the middle register of the piano keyboard, and this approximates 200, which is the negative GSP of measures 188 – 220. The arborescence at the end of section C begins at measure 207, which is the GSP.

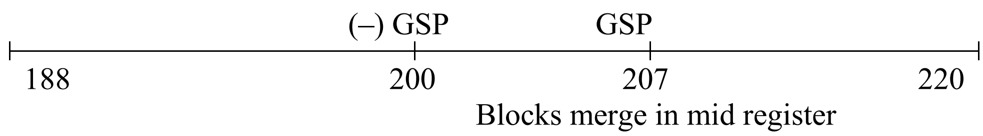

Figure 4-93: Section C, m. 188 –220

The GSP of measures 136 – 188, from the beginning of section C to the GSP of section C, is at measure 168, which is the negative GSP of section C.

The negative and positive GSPs of measures from 168, the negative GSP of section C, to measure 220 are at measures 188 and 200, respectively.

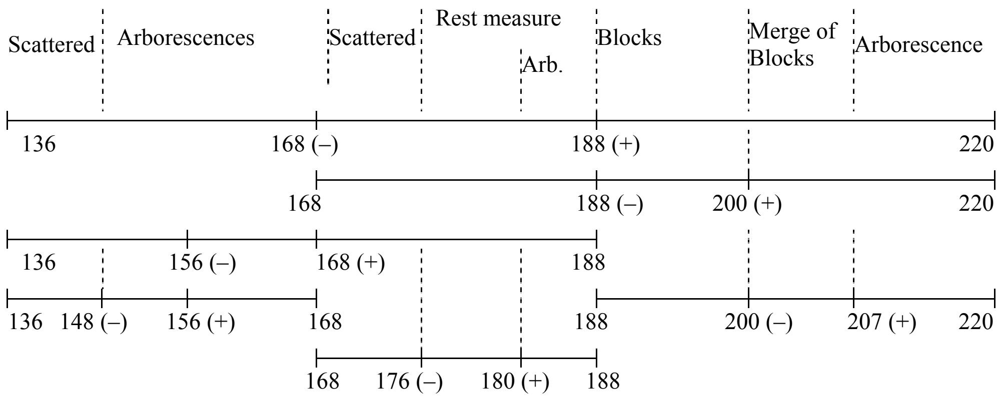

Figure 4-94: Section C, Proportion

CHAPTER 5

Implications for Performance

Many scholars and performers have questioned the possibility of playing Xenakis's pieces accurately. Accuracy is one of the biggest concerns in the performance of Xenakis's pieces because of considerable technical difficulties and physical limitations. As for his piano pieces, certain chords lie well beyond normal stretch[83] and this may well lead to a discussion on the relevance of accurate performance. In *Evryali*, not only large intervals but also incessant sixteenth notes throughout the piece add to the difficulty to it. Hill (1975) said, "each performance will become an attempt at an ideal but unrealizable perfection," about *Evryali*. Some other composers expressed doubts directly to Xenakis about the difficulty of his piano pieces for the performer and even suggested he write them for more instruments, for instance, for two pianos.[84]

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[83] Peter Hill, "Xenakis and the Performer," *Tempo* 115 (1975): 17-22.

[84] Varga, 40.

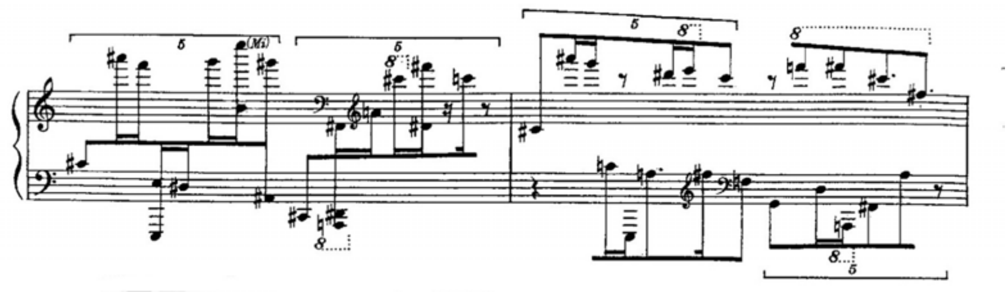

Figure 5-1:* Herma *m.64 – 65; Stretches and Leaps

Figure 5-2:* Evryali *m.84 – 85; Stretches

During his interview with Varga in 1980, Xenakis also mentioned that the composer is responsible even for the tiniest details and, based on this belief, he uses continuously changing, exact dynamics. Each line of polyphony is given a drastically different level of dynamics. Crescendos and decrescendos happen at the same time in different voices, sometimes radically from *ppp* to *fff* within a couple of notes. The coda of *Evryali* shows one of the extreme examples of dynamic change from *ff* to *mf*, to *pp*, and so on, at every sixteenth note (Figure 5-3). Juxtapositions and superimpositions of different sets or operations are distinguished by different dynamics in *Herma* (Figure 5-4). The extreme cases show that dynamics change at every sixteenth note and two completely opposite dynamics are superimposed within a chord, as the sets are occurring together. This complexity of minute details invokes the larger aesthetics of the European total serialists. Although he basically opposed that general approach, basic issues of control might be attributed to Messiaen, who was Xenakis's teacher. However, this does not mean that Xenakis's style of composition directly follows that of Messiaen's; Xenakis did not develop Messiaen's compositional principles further, although of course Xenakis learned about Messiaen's rhythmic treatment and so on. Messiaen did not try to oblige Xenakis to get through traditional music education, but encouraged him to use and combine his unique background and knowledge as an engineer/architect into composition.

Figure 5-3: **Evryali*****, Coda***

Figure 5-4: **Herma*, m.146 – 147***

Even though the case of the coda of *Evryali* contains passages which could be understood as crescendo – decrescndos over a short period of time, it is dangerous to conceive the dynamics here as linear or continuous. It is more important to understand the individually given dynamics, for Xenakis declared the meaning of the sound given by the composer is not only in the general construction but also in the minutest details.[85]

The visual complexity of the score is also a reason that contributes to the difficulty of performing of Xenakis's pieces. *Evryali*, for example, expands up to

[85] Ibid., 64.

eight voices distributed in four staves in places. The individual voices move according to arborescence: the "branches" invade each other's register at times and have independent dynamic plans.

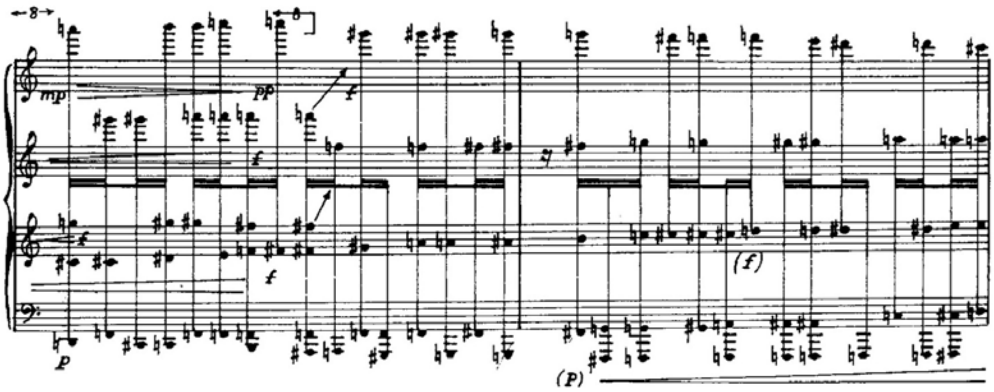

Figure 5-5:* Evryali*, m. 82

Suggestions for reduction might be made for the measures where notes are distributed beyond the reach of the two hands. However, this might lead the performer to a failure in understanding and realization of the polyphonic lines that Xenakis extended. Apparent chords are not to be interpreted as verticalities, but to be understood as part of linear polyphonic voice leading. Given the polyphonic notion, inverted chords would lose their structural continuity, especially in the case of *Evryali*. Although *Herma* does not consist of many chords, when any of the octave placements has changed, it breaks the principle of the set theory because two notes of an octave distance, that is, based on their registral disposition, are completely different elements in sets. Wide melodic intervals are more significant than harmonic intervals in *Herma*, whereas the stretches that occur in the form of chords are of concern in *Evryali*.

During an interview with Varga in 1980, Xenakis said his works are to be performed according to the score, in the required tempo and in an accurate manner.[86] In the same interview, he also said that he gives the possibility of an easier solution when necessary: he left the soloist the choice of omitting some notes or playing all of them as in *Synaphai* for piano and orchestra.[87] Therefore, it is more important to perform Xenakis's pieces as written than to modify them for playability or technical facility.

For both *Herma* and *Evryali*, awareness of each polyphonic line is crucial. Perception of one voice thoroughly and then another, and eventually adding one voice on top of another is more important than in polyphonic pieces of other composers, for Xenakis's polyphonic texture is not pianistically considered but mathematically built. The repeated notes within blocked chords in *Evryali* also do not have regular organization that could be perceived by musical intuition but are mathematically constructed. Therefore the repeated notes are to be studied

[86] Ibid., 64.

[87] Ibid., 65.

separately from the notation of other voices until the performer physically knows and remembers it.

However, some recordings of pianists who worked with the composer, demonstrate the possibility of adjusting the overall tempo slightly so it is easier to stretch and jump the span of large intervals. The performers sometimes make choices to omit the second or third repeated note(s) when they are unreachable so the voices maintain their continuity. Some other recordings also show that the performers make tempo changes when needed at times; some performances pull back the tempo towards the jumps and stretches and return to the previous tempo as the difficulty decreases and the texture lightens.

Nonetheless, it seems that the composer and some performers agree that the more important part in performance is not only the physical approach to the music but also that of the heart and the ear. Takahashi says that the ear, besides hands, recognizes the sonority of the sound cloud and its incessant changes, and listening guides the performer throughout: hand and ear collaborate.[88] Xenakis's own comment on Takahashi's performance that "he could play it [*Herma*] by heart"[89] determines the performer's approach to Xenakis's pieces.

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[88] Yuji Takahashi, "Letters to the Editor," *Tempo* 115 (1975): 53.

[89] Varga, 40.

CHAPTER 6
Epilogue

Iannis Xenakis's approach to composition and his attitude as a composer might be seen as a natural consequence of the twentieth century, the music in need of new, unexplored sources. After the dodecaphony of the Viennese school, serialism extended its realm from pitches to the other parameters of music, such as dynamics, timbre, and duration. The serial approach needed an alternative and composers started to search for new approaches and pre-compositional means.

Xenakis began exploring new sources of both sound and compositional method. He used the computer for sonic realization of composition as well as compositional aid. More specifically, mathematical calculations were computed. Some other composers of that time applied mathematical principles toward serialism as well; however, use of numbers to create series had its limitations and Xenakis expanded mathematics towards the organizing principles and those related to architecture; calculations including various stochastic distributions of sound elements in such pieces, since *Metastaseis* and *Achorripsis* are performed with the aid of a computer. His new notational system, which transfers geographic graphs of pressure-time drawn by hand and rhythmic patterns calculated on the computer into sonic events, as a part of the electronic music research center he founded, was the basis of such pieces as *Evryali*.

As has been reviewed briefly here, Xenakis's solutions to the need of new compositional sources outside of music came from mathematics and architecture. In this way Xenakis found a way to compose based on his experience as an engineer. The overall formalization of his musical works follows the structure and form of his architectural projects, in such pieces as *Metastaseis*, *Herma*, and *Evryali*. *Metastaseis* had collaborative feedbacks with the Couvent de la Tourette and the Philips Pavilion; *Herma* and *Evryali* were constructed on the golden section principle, which is one of the main structural proportions in Xenakis's architectural projects. Mathematical laws and equations, especially those of probability theory, determine the placement of actual phrases and notes. Other symbolic principles such as Boolean algebra also contributed to the compositional works of Xenakis.

Xenakis's approach to composition, especially when related to mathematical calculations, demonstrated his background as an engineer. For every piece Xenakis composed until his death in 2001, he set up mathematical or architectural reasoning and described the process distinctly. The formulae are given specifically, hypotheses are carefully examined, calculations, especially when made by hand, are given step by step, and even computer programming sources are provided. Although some other twentieth century composers might have shown connections between their music and the scientific or engineering field, the above could definitely be understood as the approach of an engineer, for a civil engineer/architect must always show every stage of calculations.

Golden section points are found at the entries of sets and their operations in *Herma*. The golden section point of the piece's large-scale structure and golden section points within smaller structures appear at the change of sets and their operations; the golden proportions of sections are related to each other, so all the proportions define the composite large. The changing densities of the referential set at the beginning of *Herma* reveal the Fibonacci series, which is closely related to golden setion, as well, while the duration of each density produces decreasing Fibonacci numbers. Numbers of sound events within sets show golden ratios as well. Not only the golden section principle but also symmetrical balance of numbers of sound events and of duration of sets or operations contributes to the structure of the piece.

Evryali also shows the golden ratio in its macro- and micro-structure. The starting point of arborescences, waves, blocks, and silences are keenly related to the golden section. Peak points of a wave, diversion of the directions in an arborescence section and others present golden sections within smaller structures.

In spite of the difficulties and the discussions on the performance of Xenakis's music, it seems that many performers find his pieces challenging, yet interesting. The mathematical and symbolic pre-compositional principles do not seem immediately related to music. However, they are bases for intellectual construction of the pieces and show inevitable associations with music. Xenakis neither expected the performer nor his own pupils to understand the mathematical constructs used to develop a piece, but he tried to draw their attention to the "core of the phenomena."[90] That is, to general understanding of his approach. With the necessary understanding of formalization in Xenakis's works, the performer may approach his music with various solutions in performance practice.

[90] Varga, 123.

Cited Compositions

Composition	Year	Publisher	Instruments	Duration
		Orchestral Music		
Metastaseis	1953-4	Boosey & Hawkes (B&H hereafter)	61 instruments	7’
Pithoprakta	1955-6	B & H	50 instruments	9’
Achorripsis	1956-7	Bote und Bock	21 instruments	7’
Syrmos	1959	Salabert	12 vn, 3 vc, 3 db	14’
Duel	1959	Salabert	A “game” for 2 orchestras	10’
ST/48	1962	B & H	48 instruments	11’
Polytope De Montreal	1967	B & H	Sound and light show, with music for 4 identical orchestras	6’
		Solo Instrument and Orchestra		
Synaphaï	1960	Salabert	Piano and orchestra	14’
		Chamber		
Analogiques A	1959	Salabert	9 strings	7’
ST/4	1962	B & H	String quartet	11’

ST/10	1962	B & H	10 instruments	11’
Atrées	1960	Salabert	10 instruments	15’
Morsima-Amorsima	1962	B & H	Piano quartet	11’

Instrumental Music for Soloists

Herma	1961	B & H	Piano	9’
Nomos Alpha	1966	B & H	Violoncello	17’
Evryali	1973	Salabert	Piano	11’
Mists	1981	Salabert	Piano	12’

Tape

Diamorphoses	1957-8	Salabert	2-track	7’
Concret PH	1958	Salabert	2-track	2’45”
Analogique B	1958-9	Salabert	2-track	7’30”
Polytope de Cluny	1972	Salabert	8-track	24’
Polytope II	1974		tape, lighting	

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